Intermediate Course

IN

Mechanical Drawing.

THORNE.

INTERMEDIATE COURSE

IN

MECHANICAL DRAWING,

BY

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My intention, in arranging this INTERMEDIATE COURSE in Mechanical Drawing, has been to present the subject of Orthographic Projection in a concise, logical and comprehensive manner, keeping theory always in view, but, at the same time, doing everything according to methods which practical experience has proven to be the easiest, most accurate, useful and readily interpreted.

The endeavor has been to avoid giving, on the one hand, a mass of definitions, rules, theorems and analyses, which are so readily forgotten, and, on the other hand, minute directions for every line, which would make the study a mere copying process; but to encourage and induce the interest, close attention and thought of the student, and thus bring about a thorough comprehension of the theory and principles and a correct training for the practice of Mechanical Drawing, so that all the apparently new conditions, which are constantly arising, can be analyzed, not by reference to text-book rules of doubtful applicability, but by the exercise of the individual reasoning powers.

WM. H. THORNE.

GOWEN AVE., MOUNT AIRY,
PHILADELPHIA, 1889.

INTRODUCTION.

As has been explained in the Junior Course, the purpose of Mechanical Drawing is to give such an illustration of a required object as to enable it to be accurately and definitely built from the drawing alone. To accomplish this purpose, the drawing must contain a sufficient number of views of the object to show the true size of every feature necessary for the information of the person who is to build it. It is not sufficient that the lines and surfaces be fully determined geometrically, but their dimensions and relative positions must also be shown in the way to be most easily understood and to require the least effort of thought in the interpretation of the drawing. The thinking must be done by the draughtsman, at least as to the form, size and purpose of the structure, and the drawing should embody that thought in every particular. Mechanical Drawing is an embodiment of thought, and, largely, of original thought, combined with technical knowledge and manual training. Hence, anything like copying should be studiously avoided when the theory and principles are being studied, and the mind should be trained from the start to work from a conception, an idea, and to put that idea on paper so as to be understood. The imagination, or what might be called the scientific imagination, must be continually on the alert, because the hand is constantly called upon to do something

that it has never done before, the different conditions and combinations which are constantly arising being infinite in their variety. Drawing from models should also be avoided. They may be used at the beginning for the purpose of giving a mental conception of the object before the student's mind has been trained to form a clear idea of it from a verbal description, or before his inventive faculties have been sufficiently developed to enable him to originate one in his own mind; but he should soon be made to realize the proper sequence of the art, which is: first, the conception, either original or derived from another; and, second, the representation of that conception by means of a drawing in such a clear, explicit and accurate manner as to enable an artisan to produce it in the concrete. The habit of drawing from models is the reverse of this, and does not tend to properly develop the imagination or train the mind for original work or produce good methods. Copying from other drawings must only be done for the purpose of gaining experience of the principles involved and the style of execution; but in every case another drawing should be made, with the proportions and conditions so changed as to compel an exercise of original thought. All drawings should be inked, and inked properly, imaginary lines being distinguished from actual lines. After spending time, work and thought on a drawing, it is simply folly to leave it in a condition which is unintelligible to any one but the author, and which will soon become so to him. The additional time required to ink, finish and dimension the drawing is amply repaid

by the clearness, permanence and usefulness of the result. Never make a drawing, not even a purely theoretical one, without using definite measurements, and always put on the essential dimensions. This habit should be cultivated from the start, as it tends to accuracy of thought and workmanship, and is a training for the proper selection of the parts requiring dimensions and of the best places to mark them.

The dimension lines and all imaginary lines, or those which do not represent parts of the object itself, but serve as bases or lines of reference, or which are necessarily used in the construction of the drawing and the preservation of which would be desirable, should be distinguished from the black lines of the object in such manner as not to interfere with the clearness of the latter. In other words, the object should stand out boldly and clearly to permit a ready general comprehension of it, while the detailed information concerning it should be kept somewhat in the background. The importance of this is not so apparent in the drawing of the simple geometrical solids in the following problems, but it is very great in all work of any intricacy.

All the devices which are used by intelligent and experienced draughtsmen to add to the usefulness, clearness and beauty of their work, should be employed by the student, in order that their use may become, in a manner, intuitive, and thus leave the mind free for concentration on the construction.

PROJECTIONS.

The Junior Course has shown that Mechanical Drawings are made upon the theory that the imaginary object is surrounded by imaginary planes perpendicular to each other, and that lines perpendicular to these planes are projected from each point of the object to each of the planes, and that the points where the projecting lines pierce the planes are the projections on these planes of the points of the object, and that these planes are then supposed to be revolved upon their intersections to bring them all into one plane,—that of the paper. The projections on each of these planes thus form a distinct view from one direction, and give the appearance which the object would present if viewed from an infinite distance in a direction perpendicular to the plane, the plane being between the object and the point of sight. The revolution of these planes into the plane of the paper brings the several views into the most convenient and sensible positions in relation to each other. It brings the Plan or Top View above, the Right Side View to the right, the Left Side View to the left, and any oblique view immediately adjacent to the part it represents. These are the positions which experience in the making of intricate drawings has proven to be the clearest and most manageable, and to be preferable to the opposite system, which imagines the object to be between the plane of projection and

the point of sight, with the result of locating the views in the opposites of these positions; that is, a view of the right side would be at the left, a view of the top would be underneath, and so on. Advocates of the latter system rarely adhere rigidly to it, and, in drawing oblique views, almost invariably violate it, and this fact is one of the strongest reasons against using it. Apart from its inconvenience, there is no vital objection to it provided that it is rigidly adhered to, because then the location of a view will immediately and positively indicate which side of the object it represents; but if the two systems are both used in the same drawing, doubt and possibly errors will result. To those acquainted with Descriptive Geometry, the method here advocated and invariably employed is the use of the third angle and not the first.

The actual indication of the planes of projection, by showing their intersections or the axes about which they are supposed to have been revolved into the plane of the paper, has already been abandoned in the *Junior Course*, and will not be used at all in this *Intermediate Course*, as it is desirable that the mind should be trained to recognize the relation of one view to another without the intervention of these axes of projection.

The number of possible views of the same object is infinite, and only such as will show in the simplest manner the essential dimensions and form should be selected. It frequently occurs, however, that one detail or unit of a structure is oblique to the main body, necessitating oblique views. Figs. 59 and 60 are

given as examples of such cases, and also for the purpose of showing the variety of different views which can be made and their proper relation to each other.

TECHNICALITIES.

Each PLATE represents a sheet of drawing paper 16 by 21 inches, with margin lines 15 by 20. The FIGURES are one-fourth size, but are to be drawn full size.

All the lines representing the OBJECT are to be inked *black*, those representing visible parts being *full* lines, and those representing hidden parts being composed of *short dots*, as shown by the full lines and short-dotted lines in the Plates. SHADE LINES are to be used in all cases, and to be located on the theory that the light falls upon the drawing from the upper left hand at an angle of 45°, and produces the same effect on all the different views, namely: of making the lower and right-hand edges shaded.

Circles and curves are to be inked first, and each shaded immediately, then all the straight *fine* and *dotted* lines, and finally the *heavy* shade lines.

After completing the black lines, ink all the CENTRE LINES or axes of symmetry and any important lines of reference, *blue:* lastly, ink all the dimension lines and any construction lines used in obtaining the lines of the object,

the preservation of which is desirable, *red.* The dimensions and the arrow-heads or points at the extremities of the dimension lines should be *black.*

In the accompanying Plates, the Figures being printed entirely in black, the blue centre lines are represented by long dots, the red construction lines by long-and-short dots, and the red dimension lines can not be mistaken. In making a drawing, however, the blue centre lines and red construction lines must not be dotted, but made full lines.

For directions as to handling instruments and materials, refer to pages 5, 6, 8, 30, 31 and 32 of the Junior Course.

Adopt some neat style of lettering that can be easily and quickly executed, and form the habit of always putting a signature and date as well as a title upon all drawings intended for practical use, in order to ensure their identification; but never make these conspicuous, because the primary object is to show the structure, to which everything should be made subservient.

Neat execution and artistic effect are desirable qualities in a drawing, but correctness, clearness, and an emphatic, unquestionable expression of precisely what is meant, is still more important. It is the beauty of the conception and the exactness with which the result can be produced in the concrete, which constitutes the beauty of a mechanical drawing, and, therefore, any attempt at scenic effect is not only in bad taste, but often detracts from its usefulness. If, however, a picture of the structure is required for the information of those who

are unable to understand a working drawing, or to conceive of what it represents, then artistic effect becomes the principal object.

OBLIQUE VIEWS.

PLATE 9.

FIG. 59. *To make a mechanical drawing of a structure which is oblique to the vertical planes of projection.*

This condition would, of course, occur only when the oblique structure was but a part of a complicated whole, the more important features of which were parallel to the planes of projection.

Let the oblique structure have a horizontal rectangular base $3\frac{1}{2}''$ long and $1\frac{3}{4}''$ wide, the longitudinal centre line of the base making an angle of 30° with the front vertical plane. Let the two sides and two ends of the structure incline at 60° with the base, and let the base, sides and ends be $\frac{1}{4}''$ thick.

In the first place, determine a convenient position on the drawing paper for the Plan or Top View, and locate the centre p of the base, through which draw the vertical line ab for the trace of a vertical plane perpendicular to the front

elevation, and the horizontal line *cd* for the trace of a vertical plane parallel to the front elevation. These two lines then represent two vertical planes perpendicular to each other and to the planes of projection, and passing through the centre of the structure, and will serve as bases from which to locate the points.

Through the central point *p*, at the given angle, 30° with *cd*, draw *gh* for the longitudinal centre line of the base, and perpendicular to *gh* draw *ef* for the transverse centre line. On these centre lines, about the central point *p*, draw the base $3\frac{1}{2}''$ by $1\frac{3}{4}''$, as given. We now have the traces of four vertical planes intersecting at the centre or axis of the structure, two of which are parallel to the planes of projection and two normal to the sides of the structure, and we have the outline of the base on a horizontal plane. Before we can obtain the Front and Side Elevations, which will be oblique views, we must first draw the projections upon planes parallel to the structure, because only upon such planes does it appear in its true form and size. Hence, project an End Elevation, *B*, of the base by drawing a line equal to its width perpendicular to *gh*, and from the extremities of this line draw the inclined sides, and, parallel with these sides and the base and $\frac{1}{4}''$ from them, draw the interior lines for their thickness.

It is a fundamental principle of mechanical drawing that each view should be made to give all the useful information than it can in the clearest manner. In the End View which we have just drawn, the additional information beyond that given by the Plan is the inclination of the sides and the thickness of the

sides and base. It is evident that this thickness can be shown more clearly by representing the structure as cut through to expose it, than by indicating it by dotted lines. The representation of such a cut is called a SECTION, and is made by equidistant fine black lines at 45° with the principal side. The distance between the lines varies from $\frac{1}{32}''$ to $\frac{1}{8}''$, according to the scale of the drawing and the size of the object, and is determined by judgment and taste. The Section is supposed to be on the central plane unless otherwise indicated, and is *always on a plane parallel to the plane of projection of the view showing it.* In the present instance, it is a Vertical Section on plane *ef*.

From this End View, *B*, and the Plan of the base, *A*, project an auxiliary Front View, *C*, by drawing the trace, $e'f'$, of the vertical plane, *ef*, and projecting the End View of the base and intersections of the sides across it, laying off half the length of the base on each side of $e'f'$, and from the extremities of the base drawing lines of the given inclination for the ends. The interior can be dotted or the view shown in Section, whichever appears most desirable. Frequently the details of a structure are such that, to insure clearness, it is necessary to make both an external view and a section on the same plane of projection, the views *C* and *D* showing their arrangement in the present instance, although in this Figure there is no necessity whatever for both.

Now complete the Plan, *A*, by projecting to it the external and internal intersections of the sides from the End View, and on these projecting lines

laying off the lengths of the intersections as obtained from the auxiliary Front View, *C* or *D* (remembering that any point is always at the same perpendicular distance from *ef* that it is from *e′f′*), and drawing the diagonal lines for the intersections of the ends with the sides.

We now have all the data for the completion of the Front and Side Elevations *E* and *F*, originally required, for which purpose draw a line perpendicular to *ab* for the horizontal base of the structure, and from this base lay off on *ab* the external and internal heights taken from view, *C*, and draw indefinite lines for these heights. Continue these lines for the heights in the Side Elevation. Project all the points down from the Plan to the Front Elevation. In the Side Elevation draw the line *c′d′* for the trace of the vertical plane *cd*, and locate all points at the same perpendicular distance from *c′d′* as they are from *cd*.

In case a Vertical Section on plane *ab* were desirable, an additional Side Elevation could be drawn in the position shown at *G*, with *c″d″* as the trace of the plane *cd*.

This Figure shows how a number of views can be projected one from the other so that, no matter how complicated or oblique a structure may be, all of its parts may be clearly and definitely shown in true porportions and proper relation, the general principle being that *all the planes of projection in a series must be perpendicular to each other*, and that *a new series of planes can be started by a plane perpendicular to either of the planes of the first series and oblique to all the rest.*

FIG. 60.—*To draw a model of a corner closet, 3″ high, with two sides perpendicular to each other, and $1\frac{3}{4}$″ wide, and a third side forming the hypothenuse of the right-angled triangle and having a central opening $1\frac{1}{2}$″ high and 1″ wide, all the sides and ends being $\frac{1}{4}$″ thick.*

This Figure is designed entirely for an exercise in projection. It is really an example of bad drawing, because more than half of the views are superfluous, and one of the requisites of a good drawing is that *it should contain no views but what are necessary to give useful information.*

Draw the Plan, *A*, with one of its square sides perpendicular to the Front Elevation, and from it project an Elevation, *B*, on a plane parallel with the inclined side. Project a Front Elevation, *C*, and a Side Elevation, *D*, as shown, making the latter a Section on the vertical plane *ab*. From the Plan project a central vertical Section, *E*, on a plane perpendicular to the inclined side, and from *E* project an oblique rear view, *F*, on a plane perpendicular to the plane of projection of *E*. From the Elevation, *B*, project an oblique side view, *G*, on a plane perpendicular to that of *B*.

Each Figure should be inked as soon as completed, and according to the directions already given—that is, the fine *black* lines and dotted lines first, and then the heavy black lines, which will complete the lines of the object. Then

ink the *blue* centre lines, *ab*, *cd*, *c'd'*, *c''d''*, *ef*, *e'f'*, and *gh*. Then ink the *red* dimension lines and put in the dimensions, points and letters in *black*.

Using Plate 9 as a guide, the student should devise other solids and draw views of them from various points, in order to become familiar and expert in treating oblique views and in obtaining projections parallel to any desired part.

DEVELOPMENTS—HEXAGONAL PRISM AND CYLINDER.

PLATE 10.

FIG. 61.—*To draw the* DEVELOPMENT *of the entire surface of a Hexagonal Prism, 2'' diameter and* $2\frac{1}{2}''$ *high, cut off at an angle of* 45° *as shown.*

First draw the projections of the Prism, and, if any difficulty is experienced, refer to Plate 7, Fig. 52, of the Junior Course, which is similar.

By the *Development* of the surface is meant unfolding it into one plane. All the sides and ends of this truncated prism are plane surfaces, but each is at an angle with the others, and the requirement is to draw them all in one plane in

their proper relative positions so that their outline could be cut out from the paper or other material on which they were drawn and the surfaces then be folded up to form the prism.

The top view shows the true width of the six vertical sides of the hexagon, therefore, take this width in the spacing dividers and step it off six times on a horizontal line, and from each of these points erect a perpendicular to this line. Imagine the surface cut on the line ab, the intersection of the two largest sides, and unfolded from that line, then ab will be the length of the extreme vertical lines $a'b'$ of the development. The line cd will be the length of the intersections of the front and rear sides with these largest sides; and if we make $c'd'$ and $c''d''$ equal to cd and draw $b'd'$ and $b''d''$, we will have these largest sides represented in their true size. In like manner, lay off the length ef on the next vertical lines of the development as $e'f'$ and $e''f''$, and gh on the central one as $g'h'$, and connect $d'f'$, $f'h'$, $h'f''$ and $f''d''$, to complete the development of the sides. Draw the hexagonal base, A, exactly as shown in the Plan, and copy the inclined top, B, from the oblique view. Every face of the solid is now drawn in its true size and in such relation to the adjoining faces that, if the figure be cut out from the paper and folded on the remaining lines, it would produce the exact form required.

This development should be repeated on card board, the outlines cut out, the other intersections cut partially to facilitate folding and the object be produced.

FIG. 62. *To draw the projections and development of a Cylinder, whose base is perpendicular to the axis and whose top is inclined.*

Let the Cylinder be 2″ diameter and the top be inclined at an angle of 45° with the axis and let the extreme height of the Cylinder be $2\frac{1}{2}$″.

A Cylinder has a curved surface generated by moving a right line so as to touch a curve, all the positions of this right line being parallel. The right line is called the *generatrix* and the curve the *directrix*. The latter may be a curve of any form, but it is only necessary to consider the case of Cylinders generated by moving a right line in contact with a circle, all the positions of the line being parallel to each other and perpendicular to the plane of the circle. If another right line be passed through the centre of the circle perpendicular to its plane it will be the *axis* of the Cylinder, and will be parallel to and equidistant from the generatrix in all its positions. The surface of the Cylinder may thus be considered as made up of an infinite number of right lines equidistant from and parallel to the axis. Again, the Cylinder may be generated by moving the circle, as a generatrix, along the right line as a directrix, always keeping the circle parallel with its first position, in which case the surface of the Cylinder may be considered as made up of an infinite number of circles the planes of which are perpendicular to the axis. Therefore, any plane perpendicular to the axis will cut the Cylinder in a circle; any plane inclined to the axis will cut it in an

ellipse, and any plane parallel to the axis will cut it right lines. Any point on the surface of a Cylinder can be definitely determined by passing through the point two planes, one perpendicular and the other parallel to the axis, for it will be at the intersection of the circle produced by the first plane with the right line produced by the second plane. A knowledge of these simple principles will enable any required projections of a Cylinder to be drawn.

In the problem under consideration, the Cylinder is 2″ diameter; hence, describe a 2″ circle for the *top view*, through the centre of which draw a vertical line *ab* and a horizontal line *cd* for the traces of two vertical planes, one perpendicular and the other parallel to the front vertical plane of projection, the intersection of which will be the axis of the Cylinder. Draw a horizontal line in the Front View for the base of the Cylinder and perpendicular projecting lines tangent to the circle in the plan to intersect this base. On one of these projecting lines lay off the given height, $2\frac{1}{2}''$, and through this point, at the given angle, 45°, draw the line of the top to intersect the other projecting line. This will complete the *front view*. Those portions of the projecting lines included between the horizontal base and the inclined top are the only visible lines produced by the curved surface alone. They are the lines in which the Cylinder would be touched by planes tangent to it and perpendicular to the plane of projection. Hence, the projection of a cylindrical surface consists of the traces of planes perpendicular to the planes of projection and tangent to the surface.

To proceed with the side view, draw the vertical line $c'd'$ for the trace of the central vertical plane cd and across it project the base, on which lay off the diameter, 2″. At the extremities of this diameter draw indefinite vertical lines for the projection of the curved surface.

As the top is inclined, its projection in the side view will be a curve, and, in order to draw this curve it will be necessary to find a sufficient number of points through which it passes by first determining these points in the top and front-views and then locating them in the side-view. To do this, divide the circle in the top-view into any number of equal parts and through each of these points draw vertical projecting lines to the front-view and also horizontal projecting lines. These projecting lines are the traces of two sets of vertical planes intersecting each other in the surface of the Cylinder. Draw them also on the side-view by laying off from $c'd'$ their distances from cd. Then, from each point where a plane intersects the projection of the inclined top in the front-view, draw a horizontal projecting line to the side-view and where this intersects the corresponding plane in that view will be the corresponding point of the curve of the top. Through all the points thus found draw a curve for the side-view of the top, which, in this case, is a circle. This will complete the side-view.

In order to find the true shape and size of the top it is necessary to project it upon a plane parallel with it. For this purpose draw, parallel with the front-view of the top, the trace $c''d''$ of the central vertical plane cd, and also the traces

of the other vertical planes at the same distance from $c''d''$ as they are from *cd*. Then draw projecting lines from all the points in the front-view, which have already been determined, to this auxiliary-view; and, where these lines intersect the corresponding vertical planes will be the positions of the points in this view.

The similarity between this Figure and the Hexagonal Prism in Fig. 61 should be noted, because the curved surface of the cylinder has been divided by lines into a number of spaces, and, as regards the lengths of these lines, the treatment is the same as if the surface was made up of that number of plane faces. These lines on the surface of the Cylinder need not be considered as the traces of planes intersecting it, but may be merely as lines at regular distances apart around the circumference. Then, if these lines are properly drawn in all the views, and a point be determined on one of the lines in one of the views, the same point can be readily found in all the other views.

To draw the development of the entire surface of this Cylinder, proceed as with hexagonal prism in Fig. 61. Draw a horizontal line, and on it lay off the length of the circumference of the Cylinder as found by calculation. Divide this length into the same number of equal parts as those in the circle in the *top view*. At each of these divisions erect a perpendicular, and on each perpendicular lay off the height as obtained from the corresponding line in the *front view*. A curve passed through these points will complete the development of the cylindrical surface. The base will be a 2″ circle the same as in the *top view*, and the top will be an ellipse the same as in the *auxiliary view*.

PYRAMIDS.

PLATE II.

FIG. 63. *To draw the development of a 2″ Hexagonal Pyramid, $2\frac{1}{2}$″ high.*

Draw the Pyramid as in Plate 7, Fig. 51, of the Junior Course.

Each side of this Pyramid is a triangle, the apex of which coincides with the apex of the Pyramid; hence, select a convenient point for a centre, and, with radius equal to the length of a side of the triangle, describe a portion of a circle, and on this step off chords the length of the base of the triangle as many times as there are triangular sides to the Pyramid, making sure to obtain the true length of both the side and the base of the triangle. Connect each of these points by right lines with the centre of the circle and with each other for the development of the inclined surfaces, and draw the hexagon of the base adjacent to the base of either of the triangles.

FIG. 64. *To draw the development of a 2″ Hexagonal Pyramid $2\frac{1}{2}$″ high, cut off by a plane parallel to and $\frac{3}{8}$″ from the axis, the plane being perpendicular to one of the sides of the base.*

First draw the projections of the entire Pyramid and its complete development, as in Fig. 63. Then cut it by the given plane and draw the projections

of the cut. From the *front view* take the length of the line *ca*, which is left by the cut, and lay this length off on the corresponding line in the development. The line *cb* is not shown in its true length in the *front view*, because it is not parallel with the plane of projection; but by drawing a horizontal line from the point *b* to a line of the Pyramid which is parallel to the plane of projection, the true length *cb′* will be obtained. Cut off the development of the base the same as in the *plan*, and transfer the vertical cut surface from the *side view* to the development, being careful to bring either of its edges in contact with the proper edge of the rest of the development.

As an *exercise*, other Prisms and Pyramids, cut in different places, should be assumed, their projections drawn, and their developments cut out of cardboard and folded up to produce the forms.

THE CONE.

PLATE 12.

FIG. 65. *To draw the projections of a Cone $2\frac{1}{2}''$ high and $2\frac{1}{4}''$ diameter at the base, cut off by a plane parallel with and $\frac{3}{8}''$ from the axis, and to draw the development of the entire surface.*

A Cone has a curved surface generated by moving a right line so as to touch a curve and at the same time to pass through a fixed point not in the plane

of the curve. The right line is the *generatrix*, the curve the *directrix*. Those cones only which have a circular directrix, and in which the generatrix passes through a point in a line perpendicular to the centre of the plane of the directrix, will be considered. This perpendicular line is the axis, the point the apex, and the cone a *right cone.*

It is evident that no right line can be drawn on the conical surface without passing through the apex, and that any plane which passes through the apex will cut the surface in right lines, if at all; also that any plane perpendicular to the axis will cut the surface in a circle the radius of which will be the perpendicular distance of any point of the circle from the axis. Hence, if it is required to determine any point on the surface, it is only necessary to pass two planes through this point, one being perpendicular to the axis and the other containing the apex.

The Cone under consideration has a base $2\frac{1}{4}''$ diameter. Locate the centre of the base in the *top-view* and through this centre draw the verticul centre-line *ab* and the horizontal centre-line *cd*, the former of which will be the trace of a vertical plane perpendicular to the front plane of projection and the latter the trace of a vertical plane parallel to the front plane of projection. The intersection of the planes *ab* and *cd* will be the axis of the cone. About this axis, describe a circle, $2\frac{1}{4}''$ diameter. Continue *ab* as *a'b'*, which will be the trace of the same vertical plane in the *front-view.* On *a'b'* lay out off the height of the cone, $2\frac{1}{2}''$,

and through the lower point thus marked off draw a horizontal line for the base. Project the extremities of the diameter of the base from the *top-view* to this line, and draw inclined lines from these points to intersect in the apex already laid off on the axis. At a convenient distance from $a'b'$ draw the vertical line $c'd'$, which will be the trace of the plane *cd* on the side vertical plane of projection. From the *front-view* project the apex and the line of the base to the *side-view* and on this line lay off the diameter of the base by marking off the radius on each side of the plane $c'd'$ which is an elevation of the plane, *cd*. Then draw the inclined lines from the extremities of the base to the apex, the same as in the *front-view*.

Having now three views of the entire Cone, it is necessary to draw the plane which is to cut it, parallel with the axis and $\frac{3}{8}''$ from it, according to the proposition. Let this cutting-plane be perpendicular to the front. Its trace will be a line parallel with and $\frac{3}{8}''$ from the axis in both top and front views, but its actual line of intersection with the conical surface will be curved and not straight. The true shape of this curve of intersection will appear in the *side-view*, and must be determined from the straight lines which constitute its projections in the *top* and *front* views. Fix upon any number of points at any distances apart on the line in the *front-view* and through them pass horizontal planes. The projections in the top-view of the intersections of these planes with the conical surface will be circles, the radius of each of which will be equal to the distance from the axis to the point where the plane cuts the conical surface. In the *top-view*, describe these

circles so as to cut the vertical cutting-plane. Draw the traces of these same horizontal planes in the *side-view* and on them lay off from the central vertical plane *c′d′* the distances from the same plane *cd* in the *top-view* of the points where the circular traces of the corresponding horizontal planes intersect the cutting-plane. A curve passed through the points thus found will be the true shape of the line of intersection of the vertical cutting-plane with the conical surface. This curve is a hyperbola.

The problem thus far could also have been solved by drawing in the three views, the traces upon the conical surface of a series of vertical planes passing through the axis and projecting the points of their intersection with the cutting-plane from the *front-view* to the corresponding traces in the *side-view;* but this method is not as accurate in this instance on account of the acute angle at which the traces intersect the plane. These traces will assist materially, however, in drawing the development of the surfaces of the Cone.

To construct this *development*, divide the circle of the base in the *top-view* into any number of equal parts, and draw diameters through these points. These diameters will be the traces of a series of vertical planes passing through the axis of the Cone. Project the extremities of these diameters to the *front* and *side-views* of the base, and draw lines from these points on the base to the apex. These lines will be the traces upon the conical surface of the same series of vertical planes, and, as the Cone is symmetrical about the axis, each line will be

of the same length and will have the same inclination to the axis as the side of the cone. Hence, describe an arc of a circle, with radius equal to the length of the side, and step off on this arc divisions of the same number and length of arc as those already made in the circle of the *top-view*, and then connect these divisions with the centre, and the result will be the *development* of the entire conical surface.

To cut away the same portion of the development as has already been done of the elevations, it is only necessary to determine in the former the location of the points which have already been fixed in the latter. These points are the intersections of the traces of a series of horizontal planes with the vertical cutting plane. Draw the traces of the horizontal planes in the development by describing arcs of radii equal to the distances from the apex to the traces on the conical surface, as shown in the *front view*. In order to locate the required points on these arcs, determine their positions in relation to the central plane *cd* in the top view by drawing lines through them from the apex to the circle of the base. Draw these same lines in the development by transferring their points from the circle in the *top view* to the arc in the *development*. Where the lines intersect the corresponding arcs in the development will be the points required, and curves drawn through the points will cut away the development to correspond with the elevations. Copy the base of the cone from the *top view*, and, adjoining it, copy the hyperbola from the *side view* to complete the development, which, if cut out

from the paper and properly rolled and folded, will produce the solid shown in the drawing.

FIG. 66. *To draw the projections of a Cone $2\frac{1}{2}''$ high and $2\frac{1}{4}''$ diameter at the base, cut off by a plane which makes an angle of 60° with the axis and which cuts the surface at a perpendicular distance of 1 inch below the apex, and to draw the development of the entire surface.*

Draw the three views of the entire cone, as in Fig. 65, and, in the *front view*, draw the trace of the cutting plane as given. Draw an auxiliary view on a plane parallel with the cutting plane.

It is first necessary to determine points on the line of intersection of the cutting plane with the conical surface in the *front view*, which can be done by intersecting it with a series of either horizontal or vertical planes, whichever can be most readily used or produce the most accurate results. It is evident that horizontal planes will intersect it at much more acute angles than vertical planes, and that the insersections will be less distinctly defined; hence it is best to use vertical planes, which, as has already been shown, must all pass through the apex of the cone. Hence, in the *top-view*, divide the circle of the base into any number of equal parts and draw diameters through these divisions for the traces of a series of vertical planes passing through the apex.

Draw the traces of these planes upon the cone in the three other views. From the points in the *front-view* where these traces intersect the trace of the cutting plane, draw projecting lines to intersect the traces in the other views. Curves drawn through the latter intersections will give the projections of the cut surface, the true size and shape of which will be given in the auxiliary-view, which is a projection on a plane parallel with this surface. The shape is an Ellipse, as is always the case when the cutting plane passes through both sides of the cone.

The surface is *developed* by describing an arc of a circle of radius equal to the length of the side of the cone, stepping off on this arc the same divisions as were used in the *top* view, connecting these divisions with the centre by lines, in order to represent upon a plane the traces already used on the conical surface, and laying off the true length of these traces, as explained in Fig. 64. A circle $2\frac{1}{4}''$ diameter and an ellipse copied from the one in the auxiliary view completes the development, which should be cut out of the paper and put into the form of the solid.

The student should draw several different Cones of different relative heights, and should cut them by vertical planes at different distances from the axis, and also by inclined planes at different angles with the axis.

PLATE 13.

FIG. 67. *To draw the projections of a Cone $2\frac{1}{2}''$ high and $2\frac{1}{4}''$ diameter at the base, cut off by a plane parallel to and $\frac{3}{8}''$ from the side.*

Draw the three views of the entire cone as before, and, in the *front view*, draw a line parallel to the side of the cone and $\frac{3}{8}''$ from it for the trace of the cutting plane. In this instance it will be best to use horizontal planes to intersect the cutting plane for the purpose of determining certain points of its intersection with the conical surface. Hence, draw the traces of any number of horizontal planes in the *front* and *side views*, and, in the top view, draw the circular traces which these planes make upon the surface of the cone. Each of these horizontal planes will draw a trace upon the cut surface, and the length of each trace is the chord of the arc of the circular trace of the same horizontal plane as shown in the *top view*. Hence, from the *front view* draw projecting lines from the points of intersection of each horizontal plane, with the cutting plane to intersect the circular traces in the *top view*, and these intersections will determine the points in relation to the central plane *cd*. On the horizontal traces in the *side view* lay off from *c′d′* on each trace the distances from *cd* in the *top view* of the points in the corresponding trace. Curves drawn through these points will complete the *top* and *side views*.

To obtain the true size and shape of the cut surface, the cone should be projected upon a plane parallel with this surface. To do this, draw $c''d''$ parallel to the trace of the cutting plane in the *front view*, for the trace in an *auxiliary view* of the central plane *cd* in the *top view*, and then draw the projections of the base and apex of the cone in this *auxiliary view*, the former of which will be an ellipse, and the latter a point. From the apex draw tangents to the ellipse to complete this view of the entire cone. Then project upon this view the traces of the horizontal planes upon the cut surface, and lay off the lengths of these traces as obtained from either the *top* or *side view*. A curve drawn through these points will give the exact shape and size of the cut surface.

This curve is a Parabola, as are all curves formed by the intersection of a right cone of any proportions with a plane parallel to its side.

FIG. 68. *To draw the projections of a Tube* $2\frac{1}{4}''$ *outside and* $1\frac{3}{4}''$ *inside diameter, cut off by a plane at an angle of* 60° *with the side, from a point* $2\frac{1}{2}''$ *from one end.*

This problem is similar to the one in Fig. 62, but is to be solved by a more rapid method, which, although not as accurate, is still sufficiently so for most practical purposes.

Draw the trace *ab* of the central vertical plane, and, in a location convenient for the top view, draw the trace *cd* of the other central vertical plane, perpendicular to the first. The intersection of these planes will be the trace of the axis

of the Tube in the *top-view*. About this point as a centre, describe a circle, $2\frac{1}{4}''$ diameter, for the exterior of the Tube, and one, $1\frac{3}{4}''$ diameter, for the interior. Draw a horizontal line for the *front-view* of the lower end, set the triangle tangent to the circles in the top-view, and draw indefinite lines up from this lower end for the front elevation of the Tube, the top being as yet undetermined. Lay off the given length $2\frac{1}{2}''$, and through this point draw the trace of the cutting plane at the given angle, 60°.

Draw, *in the side-view*, the trace $c'd'$ of the central vertical plane cd and the base and sides of the Tube, leaving the top indefinite. As the line of intersection of a Cylinder by a plane, which cuts the axis, is either a circle or an ellipse, the *side-view* of the top in this case will be two ellipses, and it is frequently allowable to approximate these ellipses by means of circular arcs, a method of finding the centres of which is given in Fig. 32, Plate 3, of the Junior Course.

From the point of intersection of the cutting-plane and the axis in the *front-view*, draw a horizontal projecting line across the side-view of the Tube. This will contain the major axes of the ellipses. Also, from the front-view, project the points where the trace of the cutting-plane intersects the exterior and interior of the Tube, to the line $c'd'$ in the *side-view*. These points will determine the minor axes. On these axes construct the approximate ellipses with circular arcs.

To obtain an *auxiliary-view* parallel with the cutting plane, draw, parallel with this plane, the trace $c''d''$ of the central vertical plane cd, and upon this

line project from the front view the external and internal extremities and the centres of the top and base. These centre lines will limit the parallel lines of the tube in this view; therefore lay off on one of them the diameters of the tube and draw the lines parallel with $c''d''$. All the points necessary for drawing approximate ellipses for the top and base in this view are now obtained.

FIG. 69. *To intersect a cone of two nappes by a plane forming an angle with the axis and cutting both nappes.*

By a cone of two nappes is meant one which is produced by a generatrix which continues beyond the apex, the result being two equal and similar cones having the same axis and the same apex, but tapering in opposite directions.

Let each cone be $4\frac{1}{4}''$ high and $3''$ diameter at the base. Draw the *top* and *front views* as shown. Draw the trace of the cutting plane in the *front view* so that it intersects the lower base $1\frac{3}{8}''$ from one extremity and the upper base $\frac{1}{4}''$ from the opposite extremity. In the *top view* draw the lines of intersection of the cutting plane with the upper and lower bases. Draw the traces of a series of horizontal planes in the *front* and *top views*. Project the points of intersection of the cutting plane with these traces in the *front view* to the corresponding traces in the *top view*, and draw the curves through the points thus obtained in

the *top view.* These two views give all necessary information, excepting the true form of the cut surfaces. These will be Hyperbolas, as is always the case when a cone is intersected by a plane which does pass through both sides and is not parallel with either.

To find the form of these surfaces, a projection must be made on a plane parallel with the cutting plane by making the trace $c'd'$ of the vertical central plane cd parallel with this cutting plane, and about $c'd'$, which, of course, contains the axis, completing the *auxiliary view* of the entire cone. Then upon this *view* project from the *front view* the traces of the intersections of the horizontal planes with the cutting plane (the lengths of these traces being obtained from the *top view*), and draw the resulting hyperbolas.

A curious fact will then become apparent, namely, that the hyperbola produced upon the upper nappe is precisely the same curve in form and size as the one on the lower nappe, and this will always occur when the same plane cuts both nappes.

INTERSECTIONS OF SOLIDS HAVING PLANE SURFACES.

PLATE 14.

FIG. 70. *To draw a vertical prism $1\frac{3}{4}''$ square and $3''$ long, intersected by a horizontal prism $1\frac{1}{2}''$ square and $4''$ long, one side of each prism to make an angle of $45°$ with the front plane of projection, and the axes to intersect at their centres.*

The importance of training the mind so as to be capable of a clear conception of the intersections of solids, will be appreciated when the fact is considered that all structures contain such intersections, which may be, and often are, the only part of the structure where any difficulty is experienced in the designing or drawing. The principles involved are really simple, and the solution of apparently complicated and difficult cases becomes easy after the habit is acquired of reducing each case into its elements.

To proceed with the present case, locate the traces of the central vertical planes *ab* and *cd* in the front, top and side views, and draw the $1\frac{3}{4}''$ square end

of the vertical prism in the top view. Lay off the height 3″ in the front view, and, midway of this height, draw the trace *ef* of a central horizontal plane in both front and side views. At the intersection of *ef* and *c′d′* draw the $1\frac{1}{2}''$ square end of the horizontal prism, and lay off the length of the latter, 4″, in the front view.

Draw the ends of the vertical prism in the front and side views and project its corners to them from the top view. Now, as the diagonal of the square of this prism is longer than that of the horizontal prism, the front and back corners of the former will not touch the latter; hence the lines of these corners will extend uninterruptedly from the top to the bottom; therefore draw these lines complete in the front and side views.

The right and left-hand corners of the vertical prism being in the same plane with the top and bottom corners of the horizontal prism, these corners will intersect, and their points of intersection are immediately obtained from the side view, as at x, the front view of which is at x' and the top view at x''. Complete the projections of these corners in all the views.

As the diagonal of the square of the horizontal prism is shorter than that of the vertical, the front and back edges of the former prism will intersect the four sides of the latter in points as yet undetermined. To find these points, project the ends of the horizontal prism from the front to the top view, lay off the length

of the diagonal on the latter, draw the corners until they intersect the sides of the vertical prism, and project these intersections y to the front-view, as y'.

The points of intersection of all the corners of the horizontal prism are now determined in all the views, but, as the sides of both prisms are inclined to the front plane of projection, it is evident that the intersection of these sides will show in the front-view. These intersections must be right lines, because the sides are planes, and the intersections of planes can only be right lines. As the lines of the corners are included in the planes of the sides, and as the points of intersections of the corners are already determined, the intersections of the sides must be right lines joining these points, as $x'y'$.

FIG. 71. *To draw a vertical prism* $1\frac{3}{4}''$ *square and* $3''$ *long, with its side at an angle of* $15°$ *with the front plane of projection, intersected by a horizontal prism* $1\frac{1}{2}''$ *square and* $4''$ *long, with its axis parallel to and its side at an angle of* $15°$ *with the front plane of projection, the axis of the horizontal prism passing* $\frac{1}{4}''$ *in front of that of the vertical prism.*

Draw the traces ab, cd, $c'd'$, ef, gh and $g'h'$ of the central, vertical and horizontal planes which contain the axes of the two prisms, cd being $\frac{1}{4}''$ in front of gh. About the intersection of ab and gh in the top view, describe the $1\frac{3}{4}''$ square end of the

vertical prism, with its side at the given angle of 15°, and about the intersection of *ef* and $g'h'$, in the side-view, draw the $1\frac{1}{2}''$ square end of the horizontal prism with its side at 15°.

Complete the projection of the horizontal prism in the top-view and that of the vertical prism in the side-view. Then the top-view will immediately give the points where the corners of the horizontal prism intersect the sides of the vertical prism, and the side-view those where the corners of the vertical intersect the sides of the horizontal. Take for instance the front right-hand corner of the vertical and the front upper corner of the horizontal prisms. The former intersects at point x in the side-view, the other projections of which are x' in the front-view and x'' in the top-view. The latter intersects at points y and z in the top-view, y' and z' in the front-view and y'' in the side-view. Lines connecting x' with y' and z', will give the lines of intersections of these sides in the front-view. Find the points of intersection of the other corners with the other sides in all the views and the lines of intersection of the sides in the front-view.

An analysis of this figure will show that in drawing it the following problems have been solved:—the intersection of vertical lines with planes perpendicular to one vertical plane of projection, but inclined to the other;—the intersection of horizontal lines with vertical planes inclined to the vertical planes of projection;—and the intersection of planes which are perpendicular to one but inclined to the other two planes of projection.

FIG. 72. *To draw a vertical hexagonal prism 2″ diameter and $3\frac{1}{2}$″ long with one side parallel to the front plane of projection, intersected by an inclined prism $1\frac{1}{2}$″ square and 4″ long, the axes intersecting at an angle of 45° at a point in the centre of the former and $1\frac{3}{4}$″ from the upper end of the latter, the sides of the latter being at 45° with the front plane of projection.*

Draw the traces *ab*, *cd* and *c′d′* of the central vertical planes, lay off on *ab* the height, $3\frac{1}{2}$″, of the hexagonal prism, draw indefinite lines for its ends, and mark the half, $1\frac{3}{4}$″, of this height, through which, at an angle of 45°, draw the trace *ef* of a plane perpendicular to the front plane of projection and containing the axis of the inclined square prism. On *ef*, from its point of intersection with *ab*, lay off $1\frac{3}{4}$″ upwards and $2\frac{1}{4}$″ downwards for the length, 4″, of the inclined prism and draw indefinite lines for its ends. In the top-view, about the intersection of the central vertical planes *ab* and *cd*, describe the 2″ hexagonal end of the vertical prism. To draw the square end of the inclined prism, select a point on *ef* through which to pass a trace *c″d″* of the central vertical plane *cd* upon a plane parallel to the end of the inclined prism. Then the plane *c″d″* must be perpendicular to *ef*, because *ef* is perpendicular to the front plane of projection, to which *cd* is parallel. About the intersection of *c″d″* and *ef*, describe the $1\frac{1}{2}$″ square end of the inclined prism with its sides at the given angle 45° with *c″d″*.

Project the extreme right-hand edge of the vertical prism from the top-view to the front-view until it intersects the end of the inclined prism, and then

project this point of intersection to the side-view and the auxiliary view, drawing this much of the edge in these views. Project the lower edge of the square prism from the auxiliary view to the front view where it will meet the right-hand edge of the vertical prism, and will determine their point of intersection, which point is then to be projected to the side-view and the edge completed. The point where the lower edge of the inclined prism emerges from the base of the vertical prism, is determined in the front-view and projected from this to the top-view. The front and back edges of the inclined prism do not touch the vertical prism, but show continuous lines in the front, top and side views, while the upper edge intersects as shown in the front-view, from which the points are projected to the top and side views. The upper square end of the inclined prism intersects two sides and the top of the vertical prism, the former points being obtained from the top-view, and the latter points from the front-view.

The front and back sides of the vertical prism intersect the sides of the inclined prism in lines determined by the auxiliary view, and from which the projections are made.

So far, every point of intersection has been directly obtained from either one or the other of the views, and this can always be done with objects whose lines are parallel with the central planes. If one branch of the object is inclined, it is only necessary to make an auxiliary view on a plane perpendicular to the central plane of the inclined branch in order to get the points of intersection

with it. Although this is not geometrically necessary, it is more accurate, and the view thus obtained is very useful in making the drawing more clear and complete, and is often necessary to enable a correct construction of the object to be made.

In order to show that the points can be obtained without the auxiliary view, the base of the vertical prism in the latter has not been completed; but its intersection with the lower sides of the inclined prism have been determined in the following manner:—

If the plane of the base of the vertical prism were continued, it would intersect the lower end of the inclined prism in a line of which the point x is the projection in the front view and *the line* $x'x''$ in the top view, and it intersects the edge of the inclined prism in a point y in the front view of which y' is the projection in the top view. Hence, $y'x'x''$ shows the projection in the top view of the intersection of the plane of the base of the vertical prism with the inclined prism, and the points where these lines cut the hexagon are the ones required.

In further illustration of this, let it be required to find the intersection of the front side of the vertical prism with the side of the inclined prism without the use of the auxiliary view. Continuing the plane of the former in the top view until it intersects the ends of the latter, project these points of intersection

to the front view and draw a line connecting them, then as much of this line as is contained within the former will be the front view of the required intersection.

It is always possible, and sometimes convenient, to find the intersection of inclined sides of objects by using only two views; but as the object of drawing is to make the construction clear, and not merely to display knowledge of geometry, the former should not be sacrificed to the latter.

In Fig. 72, the top and front views fully determine, in a geometrical sense, every point of the object, but a good mechanic would be sorely puzzled in attempting to construct it from these views alone, unless he were told that the inclined prism was to be $1\frac{1}{2}''$ square. The side view, however, could be dispensed with, as it conveys no additional information.

FIG. 73. *To draw a vertical hexagonal prism $2''$ diameter and $3\frac{1}{4}''$ long with one side parallel to the front plane of projection, intersected on one side only by another hexagonal prism $1\frac{1}{2}''$ diameter at an angle of $30°$ with the horizontal plane, the front side of both prisms to be in the same vertical plane, the planes of the axes to intersect at a distance of $1\frac{1}{2}''$ above the base of the vertical prism and the end of the inclined prism to be at a distance of $2\frac{1}{8}''$ from this intersection.*

Draw the traces of the central vertical planes *ab*, *cd* and *c′d′*, and about these construct the top, front and side views of the vertical prism. In the front view lay off on *ab* a point $1\frac{1}{2}''$ above the base, and through this, at an angle of 60° with *ab*, draw *ef* for the trace of an oblique plane containing the axis of the inclined prism. Perpendicular to this, draw *c″d″* for the trace in an auxiliary view of the vertical plane *cd*, and complete this view of the vertical prism.

As the difference of the diameters of the two prisms is $\frac{1}{2}''$, and as their front sides are in the same vertical plane, the vertical planes of their axes will be $\frac{1}{4}''$ apart. Therefore, draw the traces *gh*, *g′h′* and *g″h″* in the top, side and auxiliary views of the central vertical plane of the inclined prism. In the auxiliary view the intersection of *g″h″* and *ef* will be the axis of the inclined prism, about which draw the $1\frac{1}{2}''$ hexagon. On *ef*, lay off $2\frac{1}{8}''$ from *ab*, and at this point draw a perpendicular to *ef* for the plane of the end of the inclined prism in the front view.

The points of intersection of the edges of the inclined prism with the vertical prism can be obtained directly from the top view. Take, for instance, the bottom edge *x y*. It intersects at point *y* in the top view, which is projected down to *y′* in front view and then across to *y″* in side view; and so with all the other edges. The intersections of the edges of the vertical prism with the inclined prism are obtained from the auxiliary view.

It will be noted that in this Figure the views which are useful for constructive purposes are Top, Front and Auxilliary Views, while the Side View could be dispensed with, excepting that in actual practice it is generally useful and often important for other purposes.

PLATE 15.

FIG. 74. *To draw a vertical Pyramid 8″ high, having a triangular base 5″ long on each side, with the rear side at an angle of 15° with the front plane of projection, the Pyramid intersecting a horizontal Prism 8″ long, having triangular ends $4\frac{1}{2}$″ long on each side, with the rear side parallel with the front plane and at a distance of $1\frac{7}{8}$″ back of the axis of the Pyramid, the horizontal centre line of the Prism to be $3\frac{1}{2}$″ above the base of the Pyramid.*

Draw the three views of the vertical pyramid, commencing with the plan of its base in the top view. Also draw the three views of the horizontal prism, commencing with its end in the side view. The points of intersection can then be obtained directly from the side view, projected across to the front view and up to the top view.

It has been already stated that two views are all that are necessary to determine all the points of a solid; and, although three views are generally more convenient and desirable, yet sometimes it becomes difficult and tedious

to employ all the views necessary to obtain directly all the points of intersection. As this Figure is a good example of the intersection of inclined lines with inclined planes, it will be well to analyze it for the purpose of understanding how such points of intersection can be obtained from the top and front views only, without the use of planes of projection perpendicular to the inclined planes.

Let it be required to find where the inclined line forming the front edge of the vertical pyramid intersects the two inclined faces of the horizontal prism. It is evident that, if we intersect these inclined planes by a vertical plane containing this inclined line, the vertical plane will draw traces on the inclined planes and that these traces will contain the points of intersection, because the line is contained by the plane which makes them.

In the Figure, the trace of this vertical plane in the top view is ab, which, projected to the front view, gives $a'b'$ and $a'b''$ as the traces on the inclined faces. These traces intersect the inclined line of the front edge of the pyramid at x and x', which are the points required.

In the same manner for the left-hand edge, draw the trace cd of its vertical plane in the top view, and project this to $c'd'$ and $c'd''$ in the front view to obtain the points y and y'.

For the right-hand edge, draw the trace no in the top view, extending the limiting lines of the inclined faces far enough to obtain the intersections, and project to $n'o'$ and $n'o''$ in the front view to obtain the points z and z'.

PLATE 16.

Fig. 75. *To draw the same Pyramid and Prism as in Fig.* 74, *the Pyramid being in a similar position, but the Prism being inclined at an angle of* 60° *with the perpendicular, its central plane crossing that of the Pyramid at a height of* $4\frac{1}{2}$ *inches, and its vertical side being* $2\frac{1}{8}$ *inches back of the axis of the Pyramid.*

In this case it is doubtful which of the two methods explained with Fig. 74 is the quickest and most reliable. If it is desired to find the intersections by projection only, a complete auxiliary view, projected upon a plane parallel with the end of the prism, will be required. This would be the clearest and most easily understood by a mechanic proposing to make the object; but, for the purpose of gaining a better understanding of the descriptive geometry involved in the use of only two views, it will be best to use the latter method.

Draw the top and front views of the Pyramid complete, and, in the front view draw the centre line of the prism crossing the axis of the pyramid $4\frac{1}{2}$ inches above its base and at an angle of 60° with the perpendicular. On this centre line draw an auxiliary end view of the Prism, and from this make its top and front projections complete.

The problem now is similar to the one in Fig. 74—that is, there are the three inclined planes of the Pyramid intersecting the two inclined planes of the prism, with this difference, that the latter incline not only to the front vertical plane, but also to the side vertical plane.

As each of the planes of the Pyramid is limited by the corners, it is only necessary to find the points of intersection of these corners with the sides of the prism. Take the line of the front corner and imagine a vertical plane which contains it to be passed through the prism. Such a plane would draw the traces ab, $a'b'$ and $a'b''$ upon the sides of the prism, and the points x and x', where these traces cut the line in the front view, will necessarily be the points of intersection required, which are then projected to x'' and x''' in the top view.

A vertical plane containing the right-hand corner will draw the traces no, $n'o'$ and $n'o''$ upon the sides of the prism, but it is necessary to imagine these sides extended far enough to reach the points n and o. Where these traces intersect the edge will give the points z, z' in the front, and z'', z''' in the top views. The points of intersection y and y' of the left-hand edge are found in the same manner.

Having found the points where the inclined lines of the edges of the vertical Pyramid pierce the inclined planes of the sides of the Prism, then lines xy, xz, yz, and $x'y'$, $x'z'$, $y'z'$ will be the lines of intersections of the inclined planes of the Pyramid with the inclined planes of the Prism.

PLATE 17.

Fig. 76. *To draw a vertical Pyramid* $6\frac{1}{2}''$ *high, having a triangular base* $5\frac{5}{8}''$ *long on each side, with the rear side at an angle of* 15° *with the front plane of projection, the Pyramid intersecting a Prism inclined at an angle of* 60° *with the perpendicular, the axis of the former piercing the central plane of the latter at a height of* $3\frac{3}{4}''$ *above the base, the Prism being* 9″ *long and having triangular ends* $3\frac{1}{4}''$ *long on each side, with the rear side parallel with the front plane of projection and at a distance of* $1\frac{1}{2}''$ *behind the axis of the Pyramid.*

Draw the top and front views of the Pyramid and Prism complete.

In attempting to find, as before, the points where the front edge of the Pyramid pierces the sides of the Prism by drawing the traces on the latter of a vertical plane containing the former, it will be discovered that these traces do not reach the line, and that, therefore, the line passes entirely outside the Prism. This being the case, it follows that the front edge of the Prism must pierce the sides of the Pyramid, and therefore the traces of a plane containing this front edge must be drawn upon these sides. Either view can be selected for this purpose, whichever is more convenient or accurate. In the present instance, draw upon the top view of the sides of the Pyramid the traces of a plane, perpendicular to the front plane of projection and containing the line of the front

edge of the prism. These traces, in the top view, will be $a'b'$, $a'c'$, which intersect the line in the required points x and y in the top view, from which are projected the points x' and y' in the front view.

It will now be understood that the method of determining the point where an inclined line pierces an inclined plane is often a matter of judicious selection or of invention, and that the best possible training for such operations is to assume different plane solids in various positions, and find their intersections. Therefore, the remainder of the intersections in this Figure are required to be found without further explanation.

PLATE 18.

Fig. 77. *To draw a vertical Pyramid* $7\frac{1}{2}$ *inches high, having a triangular base* 4 *inches long on each side, with the rear side at an angle of* 15° *with the front plane of projection, the Pyramid intersecting another Pyramid* 9 *inches long, having also a* 4-*inch triangular base, the axis of the second Pyramid intersecting that of the first at an angle of* 60°, *and at a point* $3\frac{3}{8}$ *inches from the base of the second and* $4\frac{1}{2}$ *inches from the base of the first, the rear side of the second also making an angle of* 15° *with the front plane of projection.*

This is a case in which there would be no benefit whatever in making more than two views, for the reason that no two sides are parallel, and no plane of

projection could be selected which would be perpendicular to more than one of them. It is an excellent example of the penetration of inclined planes by inclined lines and of the intersection of inclined planes.

Draw the top and front views of the Pyramids complete. Then a vertical plane containing the front edge of the vertical Pyramid will draw the traces *ab* and *ac* upon the sides of the inclined Pyramid, and where these traces intersect this edge in the front view will be the points where it pierces the sides, which points are then projected to the top view.

A vertical plane containing the left-hand edge of the vertical Pyramid will draw the trace *de* upon the upper side of the inclined Pyramid, and where *de* intersects this edge in the front-view will be the point where the edge pierces the upper side. But if a trace of this same vertical plane be drawn on the lower side, it will be found not to intersect the edge; hence, the edge must pierce the rear side, the trace on which is *dn*, which does intersect the edge, giving the point where it pierces the rear side.

A vertical plane containing the right-hand edge of the vertical Pyramid will draw the traces *fg* upon the upper and *hm* upon the lower sides of the inclined Pyramid and where *fg* intersects this edge in the front-view will be the point where the edge pierces the upper side, and where *hm* intersects it will be the point on the lower side.

The points where the three edges of the vertical Pyramid pierce the sides of the inclined Pyramid being now determined, the next step is to connect these points for the lines of intersection made by the sides of the two Pyramids. The three points on the upper side of the inclined Pyramid can be readily connected; also, the points where the front and right-hand edges pierce the lower side; but a difficulty arises with the left-hand edge from the fact that it pierces the rear side, and a single line of intersection cannot be drawn on the surface of the inclined Pyramid which will connect a point on the rear side with two points on the lower side. The inference from this is, that the lower edge of the inclined Pyramid must pierce the Vertical Pyramid.

To find these points, cut the vertical Pyramid, in the front-view, by a plane perpendicular to the front plane of projection and containing the lower edge of the inclined Pyramid. This will draw the traces *op* on the rear and *ps* on the left-hand sides of the vertical Pyramid, in the top-view, and where *op* cuts the lower edge, will be the point where it pierces the rear side, and where *ps* cuts it, will be the point where it pierces the left-hand side. Then these points can be connected with those previously found to complete the intersection of the two Pyramids.

The study and analysis of the intersections of Solids with Plane Surfaces form such an excellent training for intricate and difficult problems in architectural

and engineering construction, that the importance of a complete mastery of the subject cannot be overestimated. Numerons combinations of solids of various forms should be drawn in various positions, and their intersections carefully worked out, when it will soon be found that difficulties, which at first seemed insurmountable, can be readily overcome.

INTERSECTIONS OF SOLIDS HAVING CURVED SURFACES.

PLATE 19.

Fig. 78. *To draw a vertical Cylinder $2\frac{1}{4}$ inches diameter and $2\frac{3}{4}$ inches long, intersected by two horizontal cylinders of the dimensions and in the positions shown, and to draw the development of all the surfaces.*

As explained in Plate 10, Fig. 62, any plane parallel with the axis of a cylinder will cut its surface in straight lines. By using a series of planes to trace lines on the surface of a cylinder, and finding the points where these lines pierce the intersecting cylinder and then connecting these points, the problem becomes a simple one.

In the present instance, cut the side views of the cylinders by vertical planes (the more the better), draw, in the top view, the traces of these planes upon the

surface of the horizontal cylinders, and project the points where these traces pierce the vertical cylinder to the traces of the vertical planes in the front view, for the required points of intersection. Then connect these points by straight or curved lines as required.

Take, for instance, the vertical plane *ab*, in the left-hand side view. This will draw the trace $a'a'$ in the top view, piercing the vertical Cylinder at a', which, being projected down to the front view, will intersect the traces of the same plane at the points $a''b''$, which are two points of the lines of intersection of the Cylinders. Any number of points can be determined in the same manner, and a line connecting them (in this case a straight line) will be the intersection. In the right-hand side view, the vertical plane *cd* draws the traces $c'c'$ in the top, and $c''c''$, $d''d''$ in the front views, the points of intersection c'' and d'' being projected from c'.

The problem consists, after all, in simply finding the projections of lines of different lengths, these lengths being obtained from whichever view gives them truly.

The lines already on the drawing give all the data necessary for the *development.*

Unroll the cylindrical surfaces into plane surfaces (as in Plate 10, Fig. 62), and draw upon the plane surfaces all the traces that are upon the cylindrical ones and in the same positions, and mark the points of intersection upon these traces. Connect these points by lines for the development of the intersections.

FIG. 79. *To draw a similar vertical Cylinder intersected by horizontal Cylinders of the dimensions and in the positions shown, and to draw the development of the entire surface.*

This is merely a modification of Fig. 78, and will best serve as an exercise and training if left to the ingenuity of the student without further explanation.

PLATE 20.

FIG. 80. *To draw a vertical Cone having a base $3\frac{1}{2}''$ diameter and sides at $60°$ with the base, intersected by a Cylinder $1\frac{1}{2}''$ diameter whose axis is perpendicular to the side of the Cone and intersects the axis of the latter $\frac{3}{8}$ inch above the base, the end of the Cylinder being $2\frac{1}{4}$ from this intersection, and the cone being truncated by a plane parallel to, and $2\frac{1}{8}$ inches from, its base; and to draw the development of the entire surface of the solid.*

Draw the top, front, and side views of the Cone and Cylinder, with the exception of their intersection, which is to be determined. Draw a complete auxiliary view of them on a plane perpendicular to the axis of the Cylinder. It now remains to determine their intersection. To do this, some method must be found in which the cutting of both Cylinder and Cone by the same plane will draw straight lines on the surface of each, the intersection of which lines will give a point of the intersection of the surfaces. As has already been explained, any plane which intersects a cone and at the same time passes through its apex

will draw straight lines upon its surface, and any plane which intersects a cylinder and is parallel to its axis will draw straight lines upon its surface. Hence, in the present instance, if we cut the Cylinder and Cone by a series of planes under the above conditions, the intersections of the traces of these planes on the surface of the Cylinder with those on the surface of the Cone will be points of intersection of these surfaces.

The Auxiliary View, being a projection upon a plane perpendicular to the axis of the Cylinder, presents a means of drawing the required cutting planes, because all planes perpendicular to it will be parallel to the axis of the Cylinder, and the traces of any number of such planes can be made to cut the apex *a* of the Cone. This view of the end of the Cylinder, having already been divided into equal parts to locate the traces upon the other views of the Cylinder which were used in obtaining the ellipses of its end, a convenient method will be to use these divisions as points through which to pass the new cutting planes. Hence, in this Auxiliary View, draw the trace of a plane from the apex *a* through each of these divisions. All of these planes will intersect in a line perpendicular to this plane of projection. Draw this line, *ab*, in the front view to intersect the plane of the base of the cone at *b* and project *b* to the top view.

Draw a line *cd*, tangent to the base of the cone, in the top and auxiliary views, transfer the points where the cutting planes intersect this tangent in the auxiliary view to the tangent in the top view, and connect these points with *b*,

in the top view. Then *bc*, *bd*, etc., in the top view, will be the traces of the cutting planes upon the horizontal plane of the base of the cone, and lines drawn from the apex *a* to the points where these traces intersect the circle of the base will be the traces of the cutting planes upon the surface of the Cone. Project these traces to the front view, and then their intersections with the traces of the cutting planes already drawn upon the surface of the cylinder in both top and front views will be points of the line of intersection of the surfaces of the cone and cylinder in these views. Project these points from the front view to the corresponding traces on the surface of the Cylinder in the side view for the corresponding points in that view.

This principle is applicable, no matter what may be the proportions or relations of the surfaces, and, in general, if the requirement is to find the intersection of any two solids, such solids must be cut by planes whose traces upon the surfaces of the solids will be lines, the projections of which can be readily drawn in the different views. The fact should be remembered that the intersection of the surfaces of any two solids is a line, and that, if these solids be cut by planes, such planes will draw lines on the surfaces of the solids which will meet at their intersection, if at all. The selection of the best location and arrangement of the planes is what most requires the exercise of the reasoning powers.

To find the intersection of two cones, for instance, cutting planes can be used which pass through the apices of both cones, in which case the planes

will draw straight lines on the surfaces of the cones, and the points where the lines meet will be points of the intersection. If the axes of the cones are parallel, cutting planes can be used which are perpendicular to these axes, in which case the traces upon the surfaces will be circles, and the intersections of the circles will be points of the intersection of the surfaces, and the projections of circles are almost as readily handled as those of straight lines.

In the cases of Spheres, Ellipsoids, and surfaces of revolution, cutting planes perpendicular to the axis will draw circles on the surface, and no great difficulty need be apprehended in treating them if the principles already explained are understood.

OBLIQUE AXIS OF SYMMETRY.

FIG. 81. *To draw the projections of a solid whose axis of symmetry is inclined to both planes of projection.*

Let the solid be a rectangular block 3 inches long, 1 inch wide, and ½ inch thick, and let the front projection of its axis of symmetry be inclined upwards at an angle of 60° with the horizontal plane, and the top projection inclines forwards at an angle of 45° with the front vertical plane. Draw the trace of a horizontal plane *ab*, and intersect it by the traces of two vertical planes *cd* and *ef* perpendicular to each other, and make the intersection of these traces, *h*, the lower end

of the inclined axis of symmetry. Draw the projections of this axis of sufficient length to exceed that of the block, say to g. Now, in order to draw the block of the given dimensions, it is only necessary to draw its projection on a plane parallel to the axis, and as $h'g'$ is a top view of this axis, a line $a'b'$ parallel with $h'g'$ will be the trace of the horizontal plane ab upon a plane parallel with the axis. Hence, if g' be projected to g''' at the same vertical distance from $a'b'$ as that of g from ab, then $g'''h'''$ will be a parallel view of the axis upon which the side of the block can be drawn, and from which an end view can be projected to determine the thickness. Now, draw the trace of a horizontal plane lm through the centre of the upper end of the block, then $l'm'$, at the same height above ab, will be the trace of this same plane in the front and side views. The thickness being laid off on the top view, the ends can be projected from the auxiliary view and the top view thus completed, from which the corners can be projected to the front view, their vertical distances from ab and $l'm'$ being the same as from $a'b'$ and lm. The corners are then projected from the front to the side view, their horizontal distances from ef and rs being the same as from $e'f'$ and $r's'$.

These traces of vertical and horizontal planes, ab, cd, ef, etc., are called, for brevity, *centre lines*, and are of great utility. They form the bases from which every point of an object can be definitely located, and are valuable lines of reference. They should always be inked blue, and preserved.

FIG. 82. *To draw the same rectangular block as in Fig.* 81, *with the front projection of its axis of symmetry inclining upwards at an angle of* 30° *with the horizontal plane and the top projection inclining backwards at an angle of* 30° *with the front vertical plane.*

Draw the three views of the axis, and from the top view project the axis upon a vertical plane parallel to it, as in Fig. 81, and upon this plane, and another perpendicular to it, draw the block in its true size, together with the horizontal and vertical planes of reference or centre lines. Locate these centre lines in the three original views, and complete the projections required.

The change in the inclination of the block in this Figure from that in Fig. 81 gives a good idea of the infinite variety of positions in which an object can be drawn, and of the judgment required in the treatment of oblique views. It is evident that, no matter how any line may be inclined, a vertical plane will always contain it, and it can be projected upon a plane of projection parallel to this vertical plane. Having thus the projection of the line in its true length upon a vertical plane, this can be considered the front view of a new series of views upon which the object, however complicated it may be, can be completely and readily drawn, and from these views those required can be projected.

As this is a problem of frequent occurrence in practical work, and one which makes a good test of a draughtsman's capacity, the student should draw different objects with the axes of symmetry leaning in different directions, and should not abandon the subject until attaining thorough familiarity with it.

REVIEW.

The surface of every structure is composed of plane surfaces or curved surfaces, or the two combined. The intersections of these surfaces are either straight or curved lines, and the intersections of lines are points; hence, points, lines, and surfaces in various relations and combinations constitute everything that Mechanical Drawing has to deal with.

A point has neither length, breadth, nor thickness; a line has length only; and a surface has length and breadth, but no thickness. A line may be considered as made up of points, as elements, arranged according to some law; and a surface may be considered as made up of lines, as elements, each bearing a certain relation to the one adjacent to it.

In analyzing any structure, planes of reference can be established in relation to it in any positions and in any number that may be desirable, and the traces of these planes can be drawn on several planes of projections, and the points and lines of the structure can be located on these planes of projection at the proper distances from the traces of the planes of reference, and any number of views of the structure thus be made. If the surface of the structure does not contain enough actual lines to enable it to be definitely determined, any of its

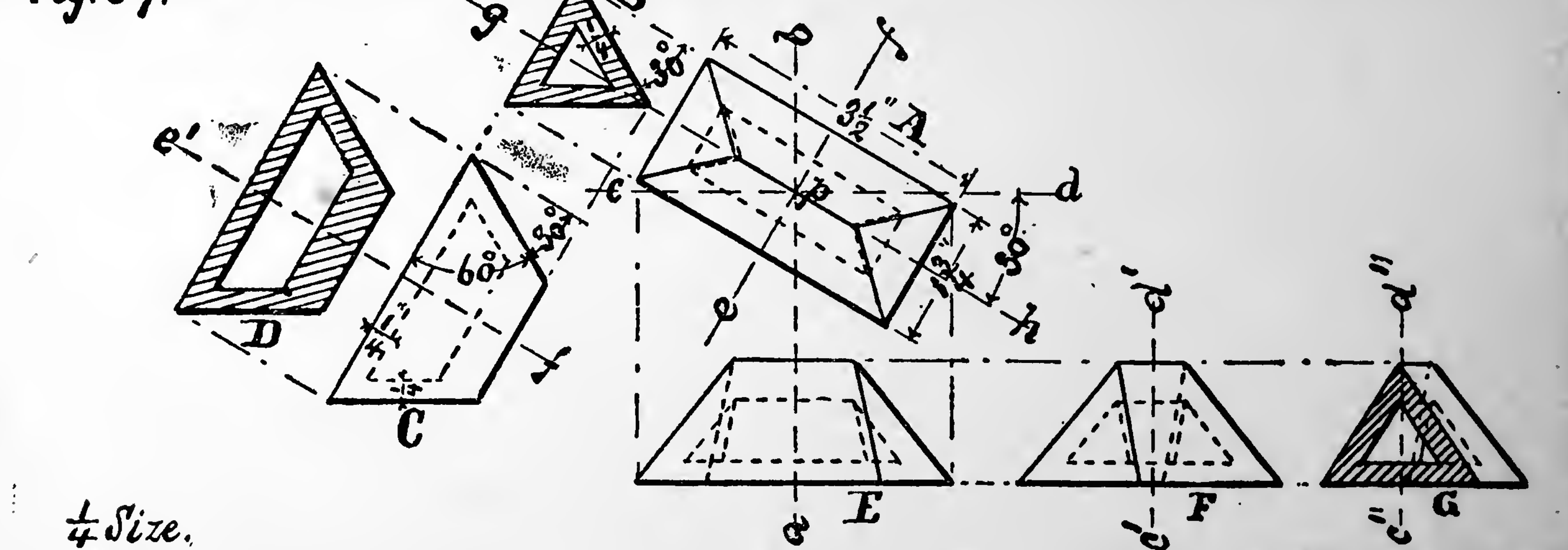
elements can be assumed to be lines and treated as such. If the lines are curved, the points where they pierce planes of reference can be determined and various projections of them be constructed.

Most structures are composed of modifications and combinations of the simple solids which have been investigated in this COURSE and the JUNIOR COURSE, and the experience and training to be obtained from a full comprehension of them should serve as a complete preparation for all the problems in orthographic projection likely to occur in practical work as applied to Engineering, Architecture, or any constructive art, with the exception of the Helix, which will be investigated in the Senior Course.

PLATE 9.

Plate 4.

Fig. 59.



¼ Size.

Fig. 60.

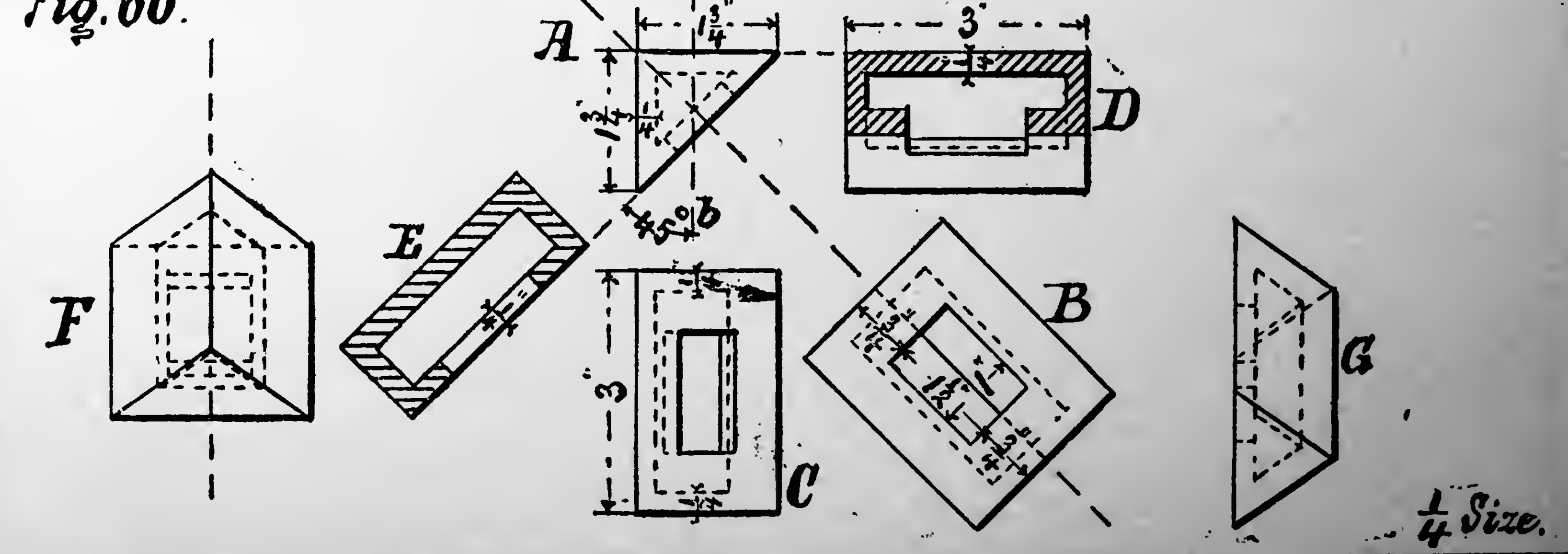


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PLATE 10.

Plate 10.

Fig. 61.

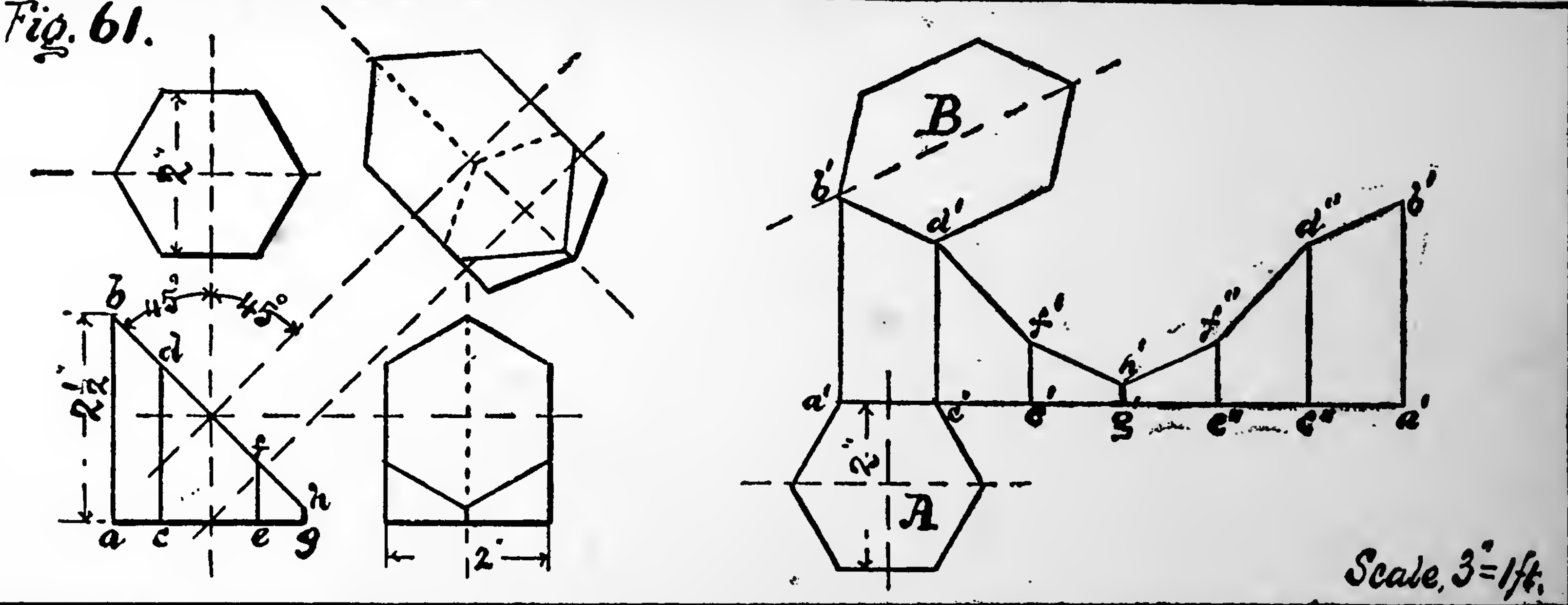


Fig. 62

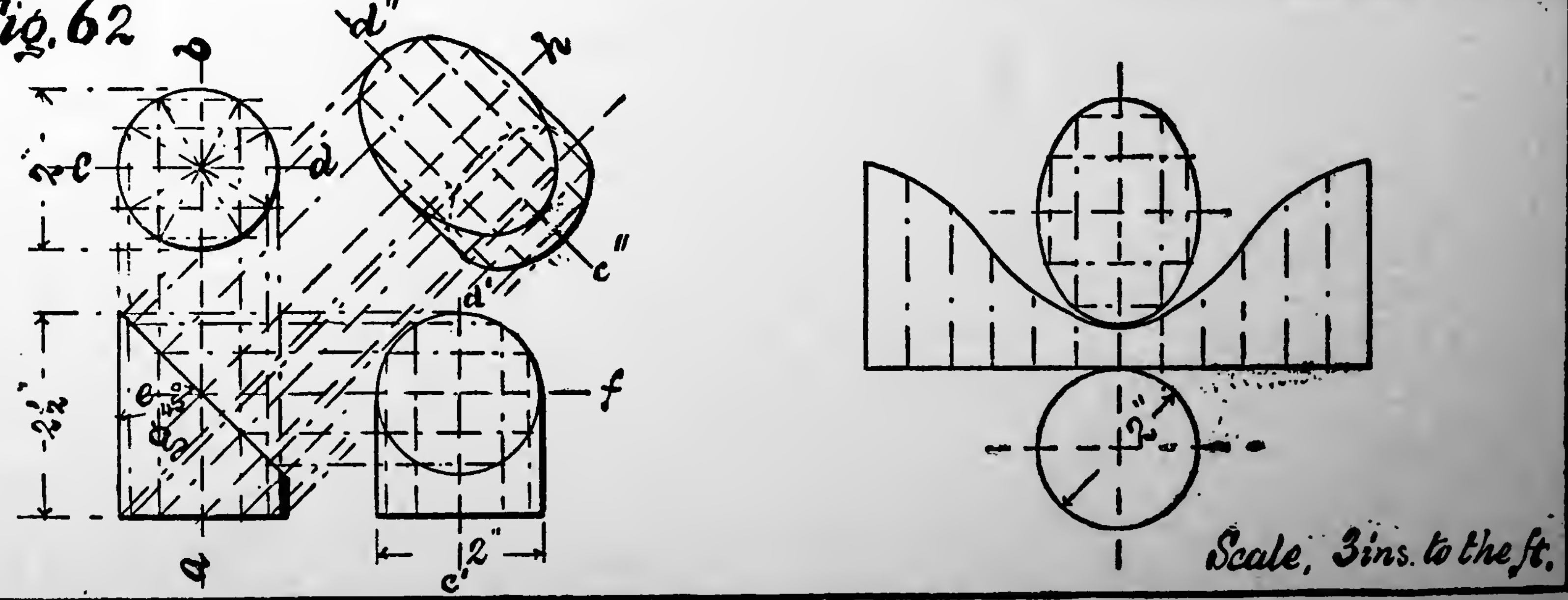
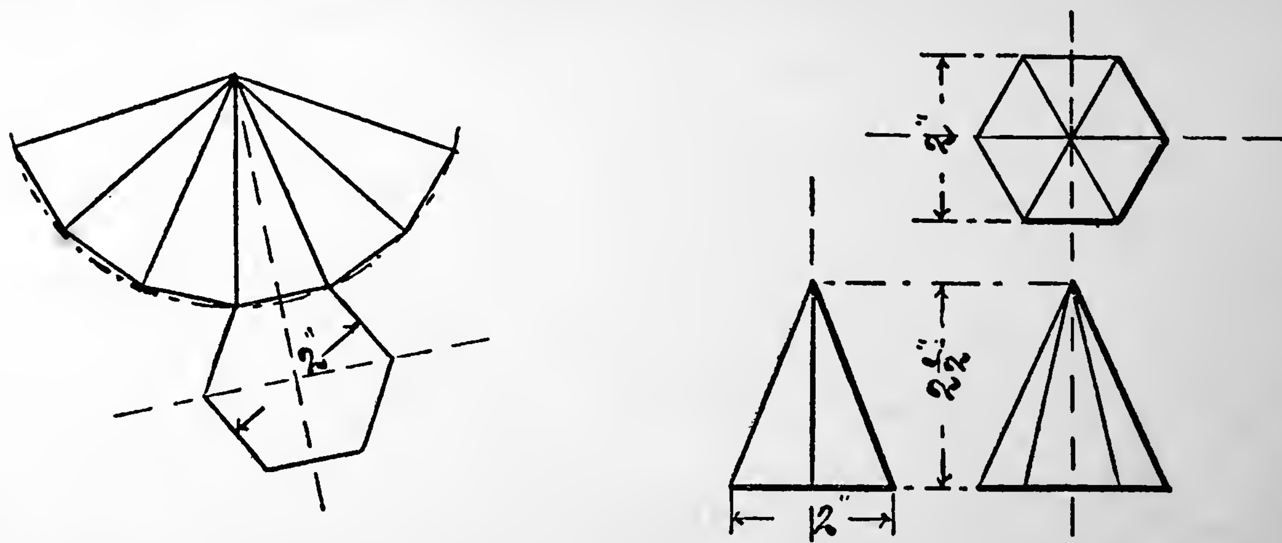


PLATE 11.

Plate 11.

Fig. 63.



Scale – 3 = 1ft.

Fig. 64.

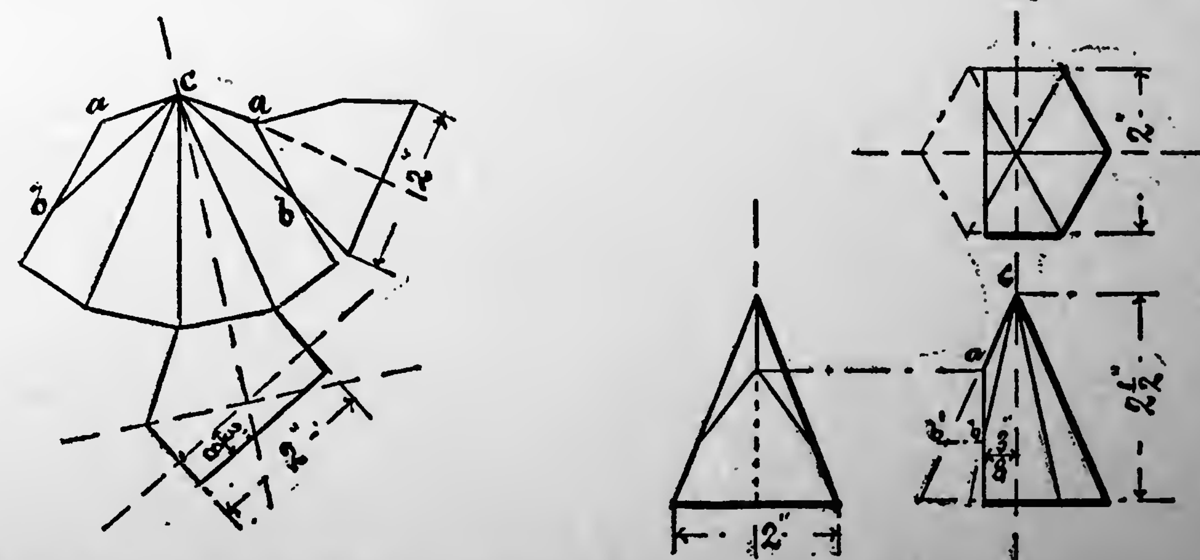


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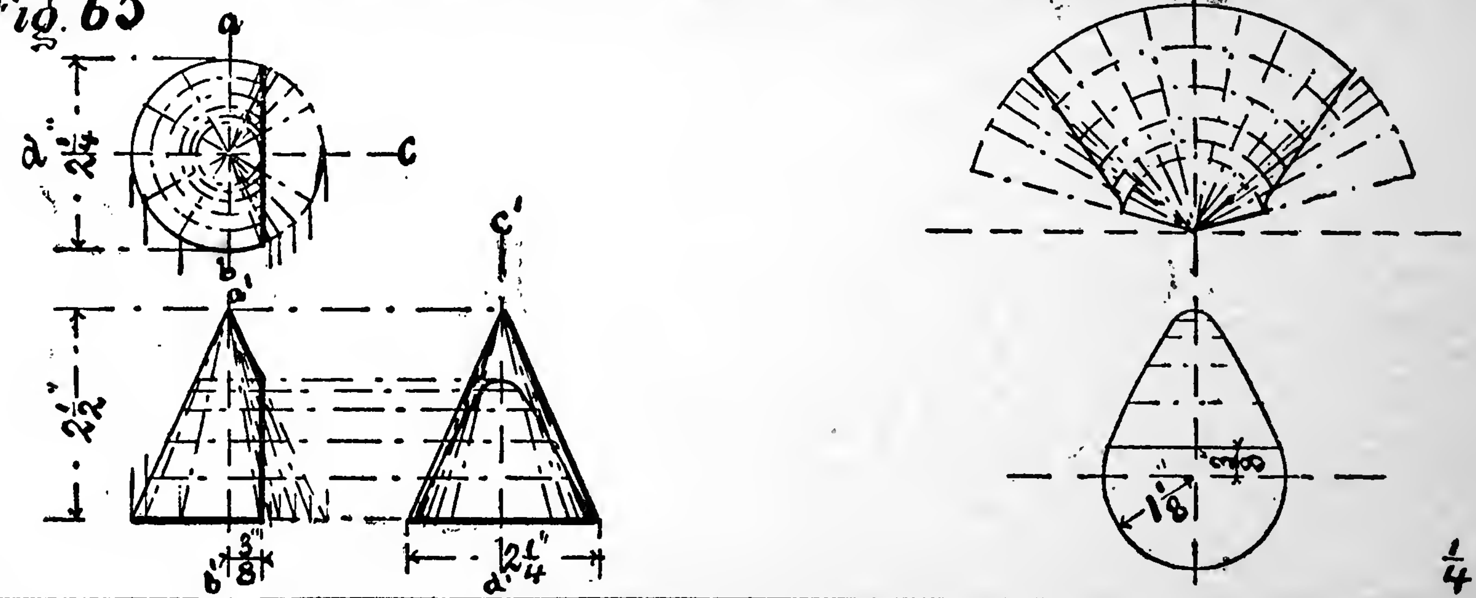
PLATE 12.

Plate 12.

Fig. 65



¼ Size.

Fig. 66.

Scale - 3" to the foot.

PLATE 13.

Plate 13.

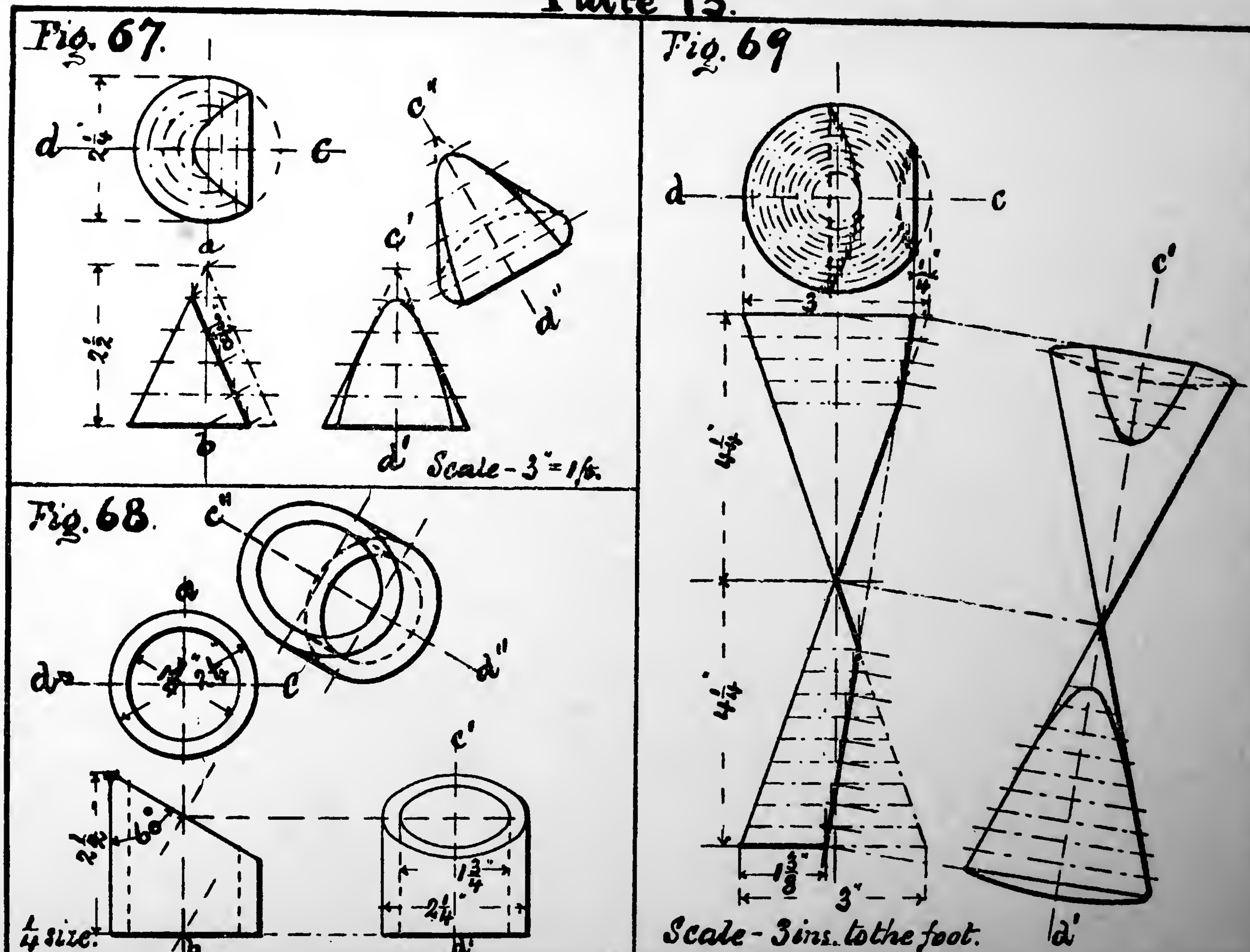


PLATE 14.

Plate 14.

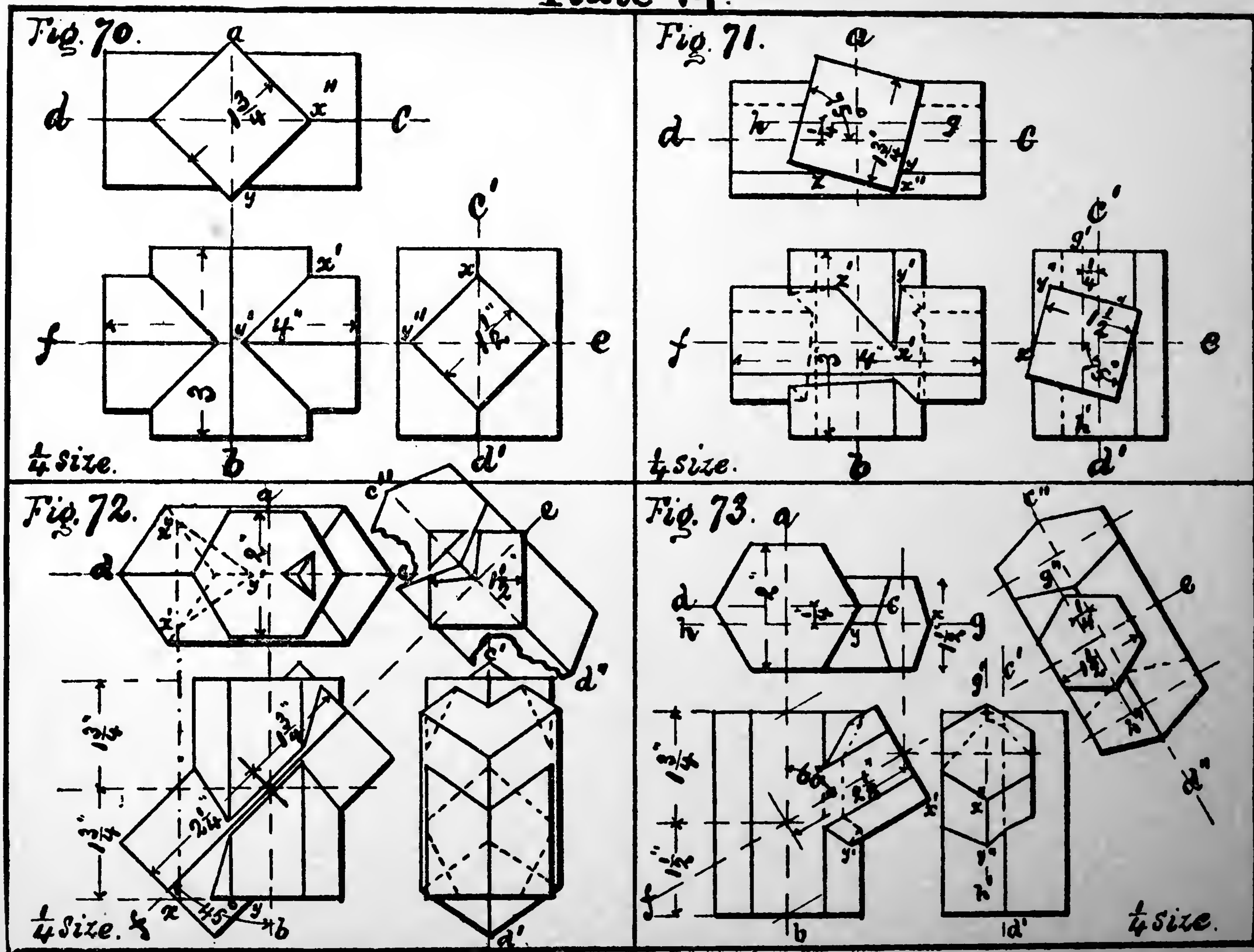


Fig. 70.
$\frac{1}{4}$ size.
Fig. 71.
$\frac{1}{4}$ size.
Fig. 72.
$\frac{1}{4}$ size.
Fig. 73.
$\frac{1}{4}$ size.

PLATE 15.

Plate 15

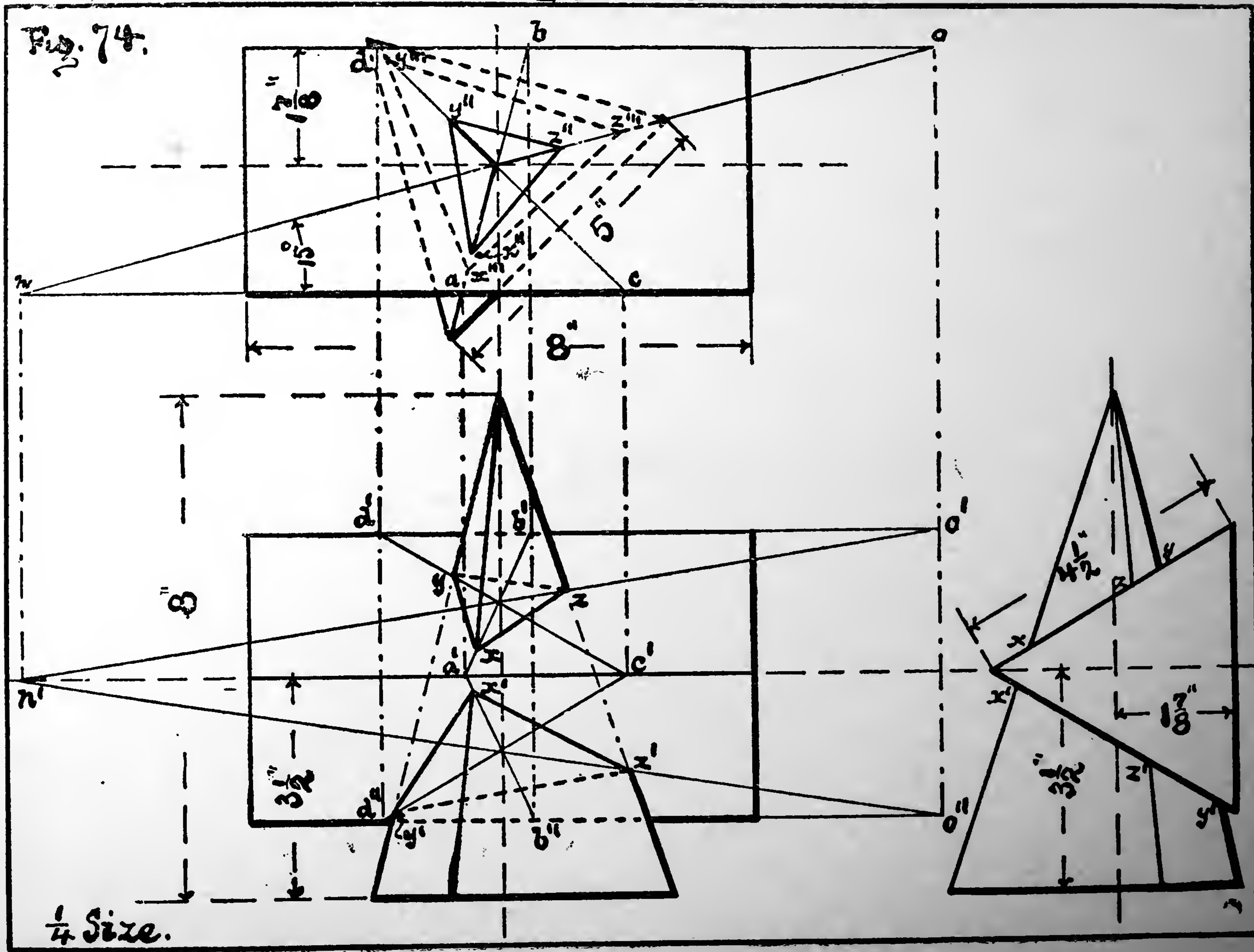
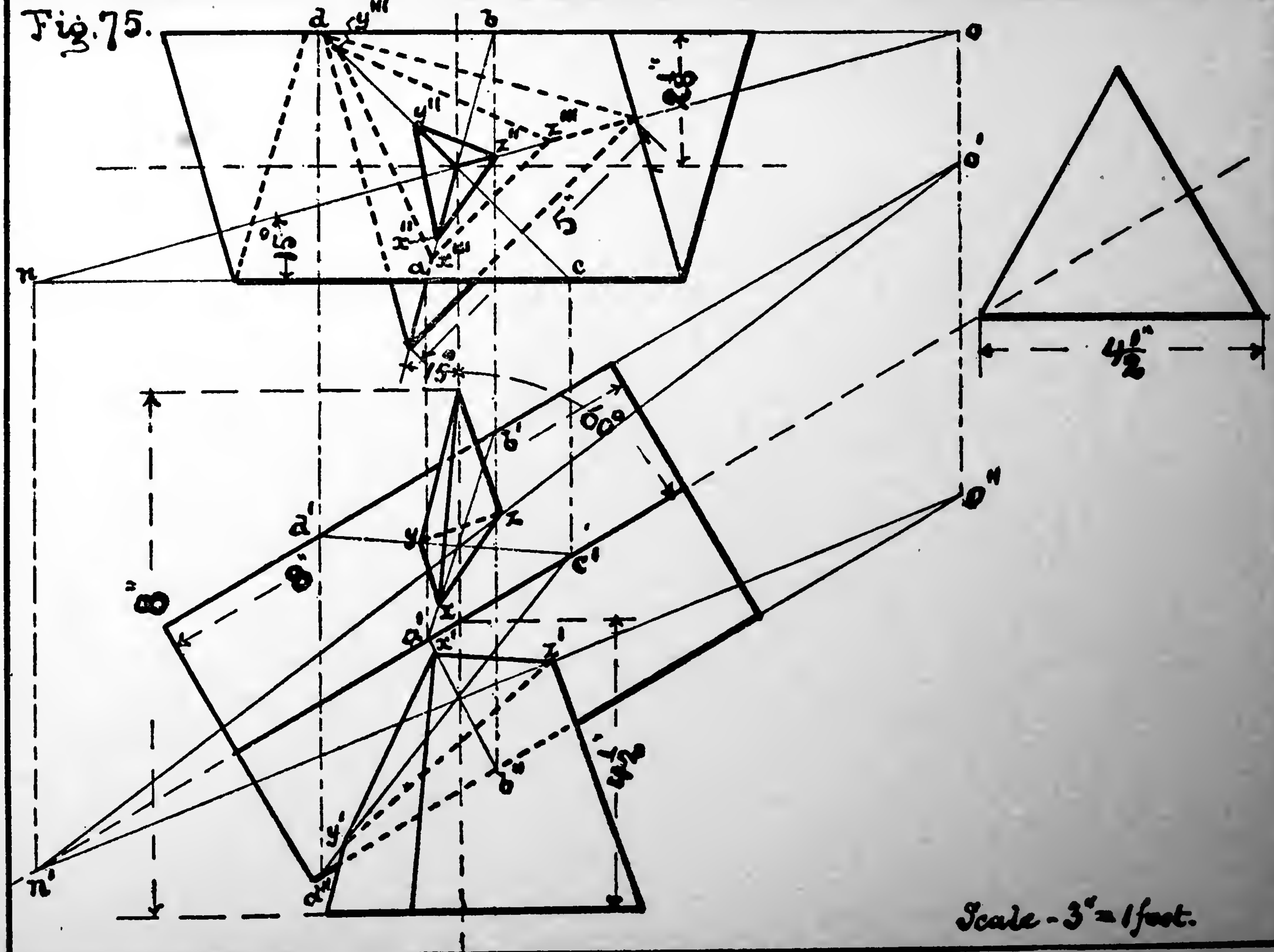


PLATE 16.

Fig. 75.



Scale - 3" = 1 foot.

PLATE 17.

Plate 17

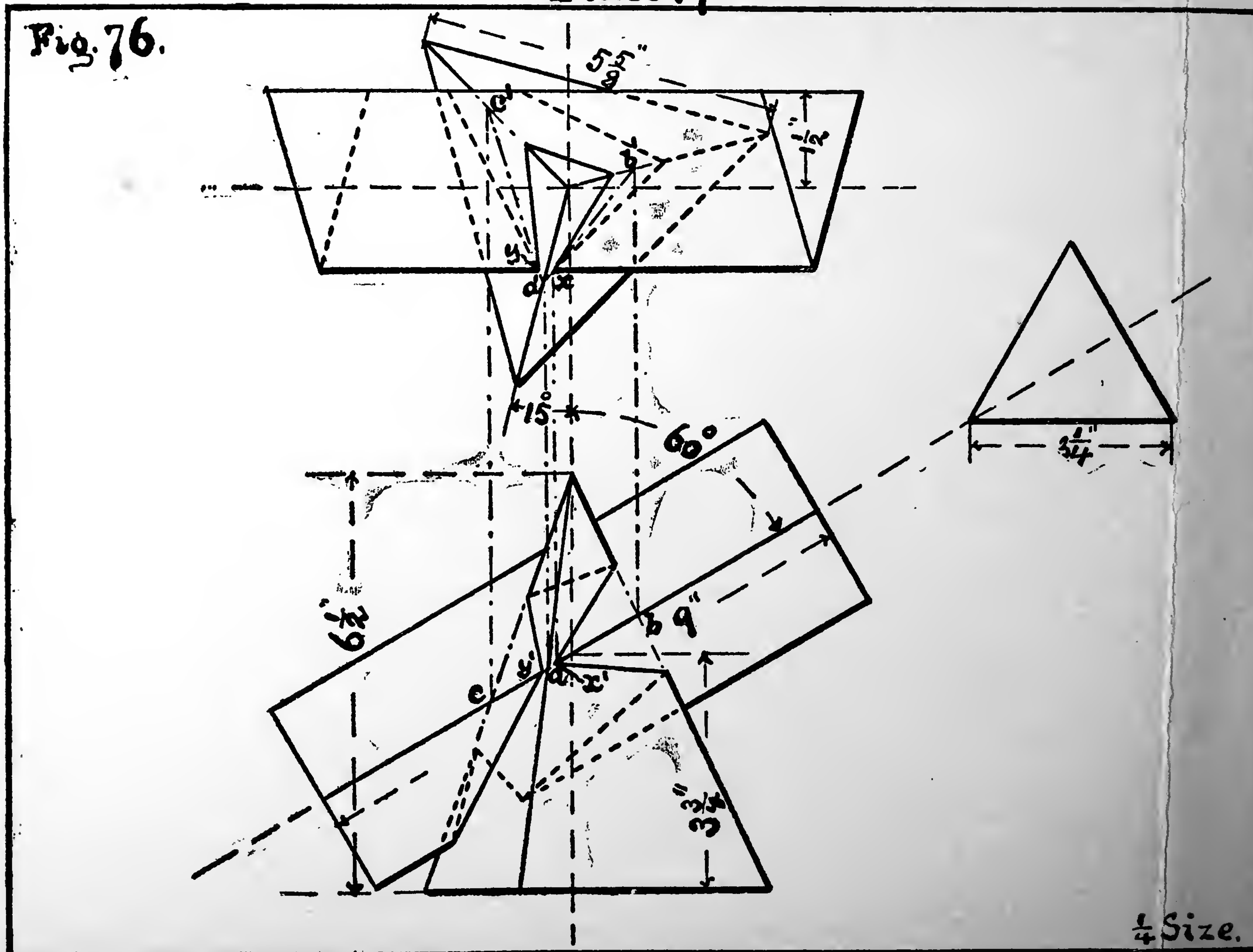


PLATE 18.

Plate 18

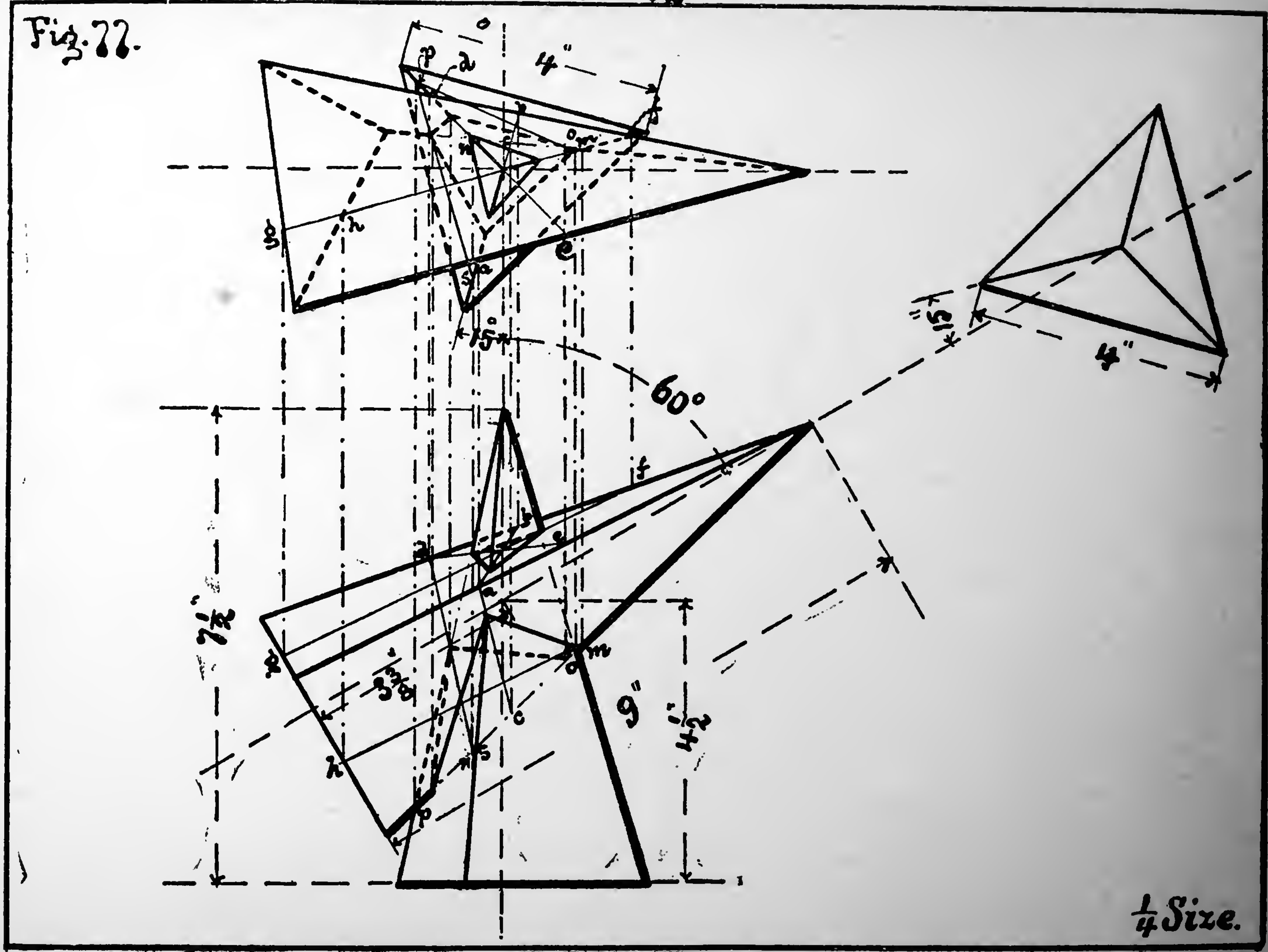


PLATE 19.

Plate 19.

Fig. 78.

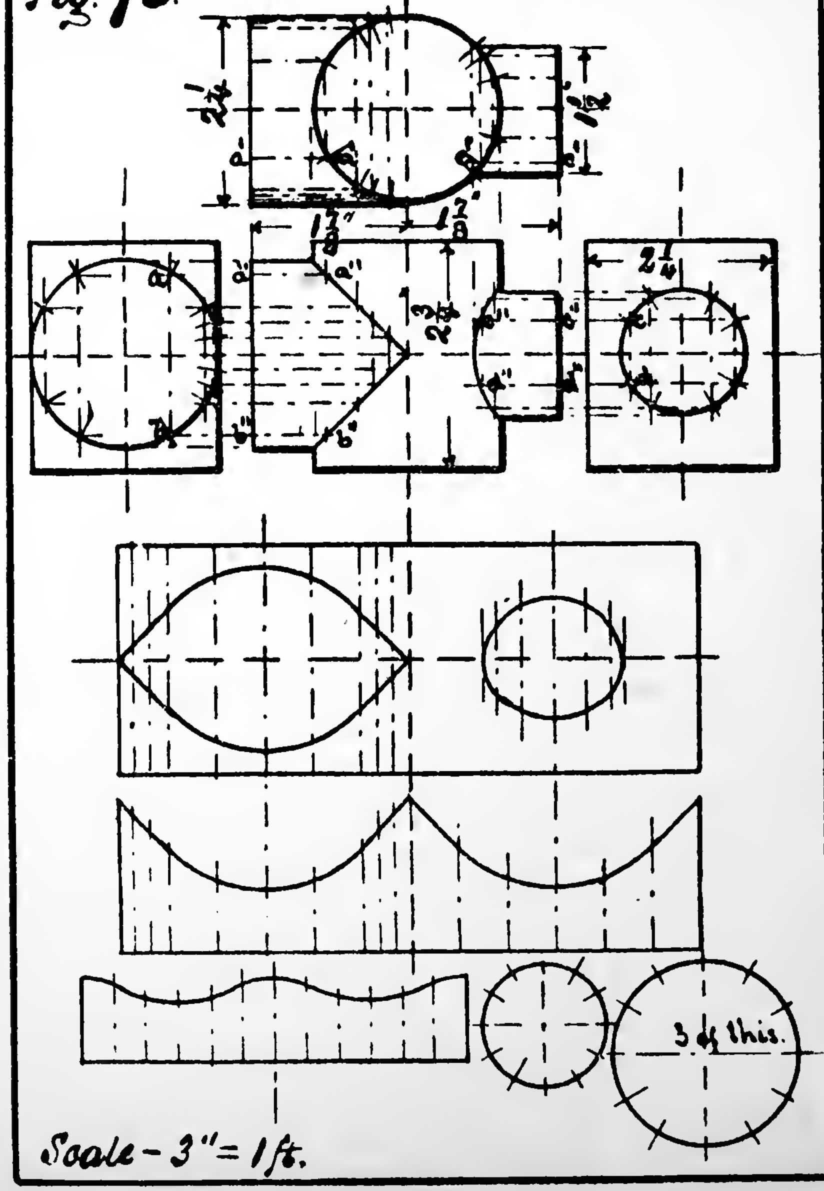


Fig. 79.

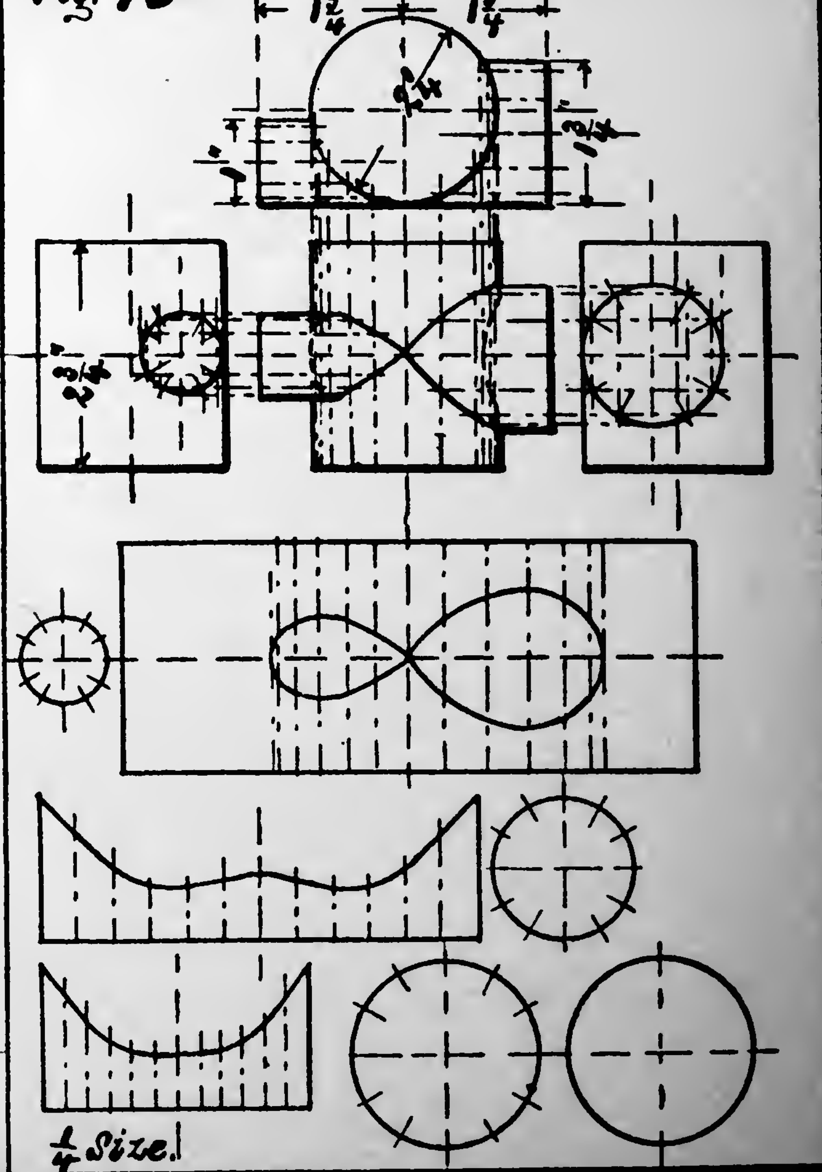


PLATE 20.

Plate 20.